TUA'LL (AND THEN) I USED MATH TO TELL A STORY: USING THINK ALOUDS TO ENHANCE AGENCY AND PROBLEM SOLVING IN AN INDIGENOUS HIGH SCHOOL MATHEMATICS CLASS

By

Jennifer Ayaginaar Boyd, B.A.

A Thesis Submitted in Partial Fulfillment of the Requirements

for the Degree of

Masters of Arts

in

Applied Linguistics

University of Alaska Fairbanks

December 2019

APPROVED:

Leslie Patterson, Ph.D., Committee Chair
Wendy Martelle, Ph.D., Committee Member
Sabine Siekmann Ph.D., Committee Member
Patrick Marlow, Program Chair
Linguistics Program
Todd Sherman, Dean
College of Liberal Arts
Michael Castellini,
Dean of the Graduate School

Abstract

This paper examines action research in a high school math classroom with a focus on student discourse and agency. Students' use of language to explain their problem-solving processes was documented and analyzed. Specifically, the focus was on variations in student language and how the teacher responded to students during the problem-solving process. The following questions guided the analysis of what happened in the classroom: 1) How do my students talk about their math process? 2) How do I mediate their problem solving? One of the teacher researcher's earliest realizations was that she interfered in students' opportunities to problem solve on their own. Additionally, the students' explanations of their "problem-solving process" included more narration than justification or explanation of the process. On reflection, the teacher researcher decided to return to the research process to look further into these interactions while students were problem-solving. The second phase of research focused on student agency and how teachers can mediate for their students. Over a four-week period, the teacher researcher looked at the influences of multiple levels of assistance while each student was talking through his or her problem-solving process. Data sources include field notes, student artifacts, videos of student think aloud videos, and transcriptions of group work from the teacher researcher's classroom. The findings provide detailed insights into how these high school students approach math problems and how they describe and explain their problem-solving processes. The teacher researcher explores implications for teacher actions, insights into how students work together, and observations of students discussing their problem solving. Specifically, the teacher researcher noticed a need for language focus in mathematics instruction. In addition, teachers should problem solve with their students, rather than for their students; and allow students to mediate with each other to develop student agency.

Table of Contents

List of Figures

Page

List of Tables

Page

List of Excerpts

List of Appendices

Chapter 1: Introduction

While chaperoning our school's basketball team to a State Tournament, I was sitting in the crowd, watching my students take on a team from a much larger, much more white community. One of the fans of our opponent's team made the comment that, "they're not much about the education in Toksook Bay," while using the best "hick" voice he could muster. I was devastated as it reminded me about all the other times someone told me that our students do not measure up. At first I was sad about what this "said" about me as a teacher, then I thought about how often my students must have to put up with comments like that.

With all the stereotypes that my students face and the fact that our school never seems to have enough funding to give them equal opportunities, it becomes my job to do everything I can to give my students a level playing field. Helping them become confident and independent learners is one way to do that. Through current theories and my own research that focuses on individual students, I want to help change the way that my students think of themselves as learners and as novice mathematicians.

In Toksook Bay, where I teach, we have been told for several years that we have the lowest test scores in the district and that we are not making the gains we need to be. When I got to Toksook, there was a letter from the previous math teacher telling me to avoid requiring students to solve word problems—that the students do not "get" them and that they are a waste of time. I know my students have heard these and other observations about their capabilities and their potential, and I cannot imagine what it has done to their self-esteem. I have been told that my students have not been taught to problem solve or think critically. Because our district has a high amount of teacher turnover, students do not typically get the opportunity to prove that they are capable of academic success. They have been taught that if they slack off a little bit or come across any bumps, people will give up on them, assume they are not capable, and do it for them.

My students are Yup'ik Native Alaskans. We live in the southwest corner of Alaska, and the community is very strong in their culture. My students spend a lot of time listening to elders, dancing traditional yuraq dances, speaking in Yugtun, participating in a subsistence lifestyle, and working on cultural crafts. In school, they are good at noticing patterns, they want to learn, and they are eager to help each other. They are held back by low expectations, low teacher retention, and a great deal of social-emotional stressors. The fact that they are learning two languages on top of all of this shows their capabilities.

My students have a lot of hardships, but in a lot of ways they are very similar to students across the nation. When math is hard in high school, it is typically because of the language component. A word problem with multiple steps--words they have to translate from English to math symbols--and complicated procedures are dismissed by students as too difficult because the problems take time to get used to with the additional language of math. Because my students have a strong desire to be right every time, math can be disheartening because it is so easy to get something wrong. I want help them think of math as a critical thinking class instead of a class where they learn a set of steps to follow on any problem. I want them to use their critical thinking to think of language as a tool for solving their math problems.

As a secondary math teacher, imagine a time when a student asks for help on a problem, but they have no idea what specifically they need help on. Other times, the student says they need help, but once told to try the problem, they are able to complete the entire problem by themselves. It seems like students just want someone to watch them solve the problem to make sure they are doing it correctly. They are afraid of failure in any form. In a regular classroom, it is unrealistic for the teacher to watch every problem that every student completes. Because of this, I want to help my students become more comfortable with their math problem solving and being able to talk through the steps of a problem. I want them to be able to articulate specific questions such as, "should I divide here since I want to undo the multiplication?" I also want them to develop more self-confidence when they are solving problems and understand that it is okay to make mistakes.

The educators who developed the Common Core also focus on critical thinking. They have developed a list of mathematical practices that they believe all students should work on and have the ability to do, despite their grade level. Some of these practices support my desire for students to be able to use language and reasoning to support their mathematical knowledge. Out of the eight practices, these are the mathematical practices (M.P.) that ask students to reason: M.P. 1) Make sense of problems and persevere in solving them and M.P. 3) Construct viable arguments and critique the reasoning of others (http://www.corestandards.org/Math/Practice/). These mathematical practices are critical if students are going to analyze problems and trust in themselves to solve the problems.

Throughout my time in the Literacy for Emergent Bilinguals program, I have thought a lot about math, the language involved, and how to best help my students learn both the concepts of math and the language of mathematicians. I have thought a lot about activities that could help my students learn the language of math, and I was entranced by the thought of letting my students work together in whatever language was comfortable for them. I struggled (and still struggle) to come up with meaningful activities for my classroom, but the integration of math and language is always in the back of my head as I lesson plan.

I thought about focusing on word problems, or zeroing in on how students communicate with each other. I thought about teaching my students the concept of the "language in math." I thought about focusing on the importance of word problems and critical thinking and how students can use language to do mathematical problem solving. I spent a semester focusing on teaching students how to analyze words to discover meanings and how to put definitions in their own words to increase comprehension of vocabulary. I thought about spending a lot of time on inquiry-based projects/questions while trying to decrease my help or involvement. I wanted the students to develop a "toolbox" full of problem-solving techniques that they would be ready to use whenever they got stumped on a problem. This program has made me passionate about a lot of possible solutions to help students learn better.

It was really hard for me to define one aspect of teaching that I could explore for my final research project. My research questions and focus changed multiple times as I tried to narrow down what I thought needed to be explored most for my students. I started to realize that no matter what the main focus was, my biggest frustration with teaching was how dependent my students were on my help. Whether I had them work on a big project with lots of steps and critical thinking, or I asked them to solve one problem that followed the same general steps that I had been drilling, I would always have at least one student who would not write anything on their paper without asking for help. I wanted my students to develop their drive to take academic risks – to try something for a project or a problem that might be scary or might not work out. I wanted them to be okay with putting all of their effort into something even if it might not be the right solution. Because of this, I decided to look into these questions: How do my students talk about their math process? And how do I mediate their problem solving?

I refuse to believe that my students are incapable of meeting the challenges that other students meet successfully. I have seen their curiosity and insights, and I know they can be pushed harder than they have been if given the right resources and support. There are always going to be people in the world who think they are better than everyone else. However, as educators, it is important for us to continue to teach our students, believing they can prove those people wrong. Our students will always face negative stereotypes and be judged for where they come from and the color of their skin. However, instead of letting bullies change us and reduce what we believe we can accomplish, we have to teach our students to let negative viewpoints motivate them. We have to teach our emerging bilingual students that their culture makes them strong learners, not weak. We have to show them that the world is changing—research is being done to figure out better ways to teach bilingual students, because being bilingual is a blessing, not a curse.

Throughout this paper I will explore what I have discovered about how students talk about their math problem solving and about how teachers can support them. In Chapter 2, I will discuss the literature that has been published on these topics and how it relates to what I did in my classroom. In Chapter 3, I will turn my focus specifically to my research, how I set it up, and the theories that shaped the decisions I made. In Chapter 4, I will talk about how I analyzed the data that I collected for my research and what I found from those data. In Chapter 5, I will talk about what I think this means for future teachers, including myself, and for future research that I think would be interesting, based on what I found from my research.

Chapter 2: Literature Review

Part of what makes us human is the search for understanding. Teachers fulfill the crucial role of guiding students in this lifelong journey. Students can make meaning with careful support from their teachers. Students are naturally diverse individuals and their learning needs are important for teachers as they decide how to address individual needs in lesson plans, how they interact with students, and what resources they use in their classrooms. Every student learns differently, and there are many theories about how students make meaning and how teachers can assist them in their learning. For my students, I focused on how they make meaning as budding mathematicians and how I could support their progress.

In this chapter (see Figure 2.1), I talk about how students make meaning and how this process means they are active meaning makers, not passive. I discuss the teacher's role in their students' meaning making. Since student meaning making is so important, teachers need to play a role in encouraging and developing that meaning making. Finally, I talk about how teachers

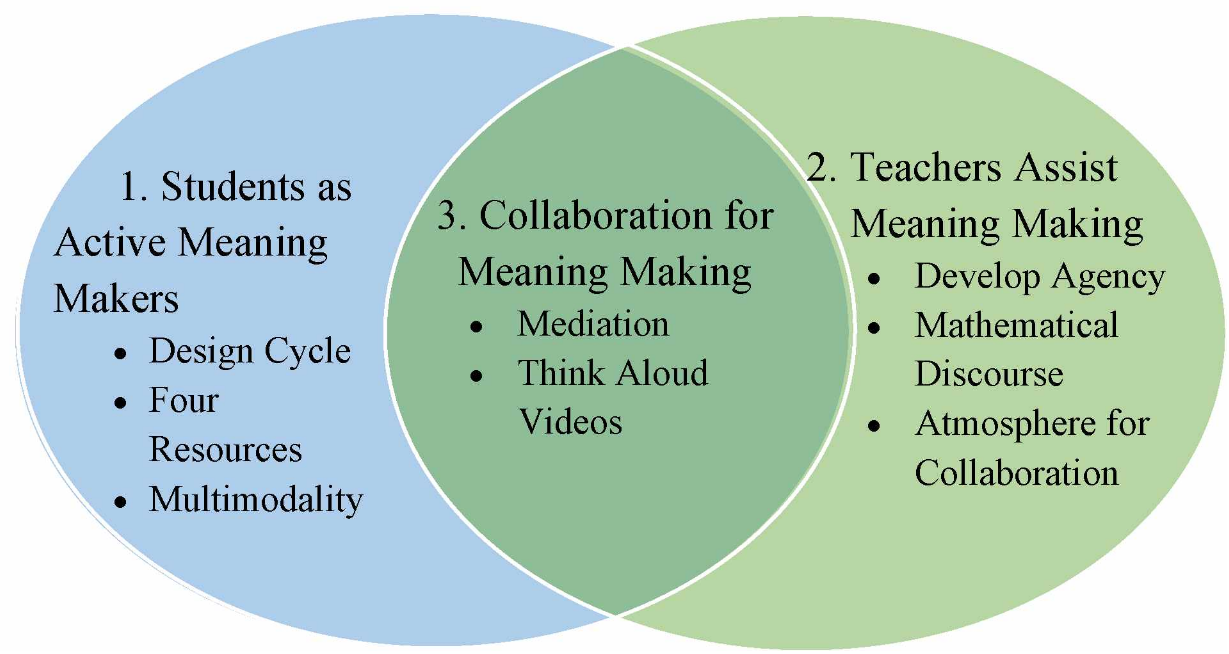


Figure 2.1: Sections of literature review: Three interdependent areas of research that help teachers invite and support students in reading, writing, and thinking as mathematicians.

and students should come together to make meaning. These concepts and processes can be applied to any subject as teachers work to develop their students into mathematician, historians, or scientists. Although all of these aspects are important for all teachers, I focus each section in the implications for math teaching and learning.

How Students Make Meaning: Learners Are Active Meaning Makers

The theoretical approaches and models that I will talk about are the design cycle, four resources model, and multimodality. Each of these perspectives provide insights into how students make meaning. They all focus on students as active meaning makers, the diversity of students, and the need for students to interact with the material in different ways. In the design cycle, students gain knowledge in a cyclical path, not a linear one. In the four resources model, students take on multiple roles when interacting with a source of information. The multimodality framework is part of a larger framework that says, among other things, that students should be introduced to new information through multiple avenues to deepen their understanding.

As active meaning makers, students need time to learn the material and make connections in diverse ways. Teachers have to look out for the whole child regardless of lesson content. Cope and Kalantzis (2009) offer a definition of meaning making that is particularly useful to math teachers because it highlights the four thinking processes that are parts of the learning cycle: experiencing, conceptualizing, analyzing, and applying. Almost all cycles of learning have similar processes, but discuss them in different ways. Teachers should weave these thinking processes throughout lessons so students can experience learning through different modes, conceptualize it to figure out what it means to them, analyze it to see if it will integrate into their perception of the world, and apply their new understanding to other contents and aspects of their lives. Cope and Kalantzis (2009) tell us that this process is not linear, but students should be allowed to move back and forth throughout the four kinds of thinking to weave their learning together.

With the Common Core State Standards increasing the problem solving and application aspects of math, today's teachers are being asked to teach math in ways they never learned (http://www.corestandards.org/Math/Practice/). In the math classes I grew up in, teachers focused mostly on experiencing and conceptualizing, less on analyzing and applying. Teachers have led students through examples so they can experience the material, then allow them to conceptualize or generalize that knowledge so they can use the same steps on slightly different problems. Every once in a while, students are asked to apply their math knowledge to a word problem, but these are typically very structured and follow the same topic that they just learned. As the focus on analyzing and applying becomes more pronounced than it has in the past, students will be required to develop language as a thinking tool. This focus on language is new for teachers and students alike, but once mastered, has the potential to let students be problem solvers instead of problem performers.

In order to develop into these adept problem solvers, students need a developing competence in the use of language. Vygotsky's socio-cultural theory assumes that language is a tool for meaning making. "Within this framework, humans are understood to utilize existing, and to create new, cultural artifacts that allow them to regulate, or more fully monitor and control, their behavior" (Lantolf, Thorne, & Poehner, 2015, p. 207). While physical tools mediate our actions on the physical world (a hammer and a nail are used to hold pieces of wood together), semiotic tools mediate our thinking and learning. Using language to discuss, argue, reason, and process allows students to become critical thinkers and gives them ownership of their knowledge. With socio-cultural theory, teachers do not fill their students with knowledge, but students are able to use their language to create and mediate their own knowledge. When students are bilingual, the various language and dialects become relevant as well.

Learners engage in the design cycle as they solve math problems.

While students are learning, through multiple modes or just one, they tend to make meaning through a cycle. The design cycle as described in Cope and Kalantzis (2009) involves available designs, designing, and the redesigned. This cycle tends to start at the available designs, but from there can go to any stage any number of times (see Figure 2.2). "What the

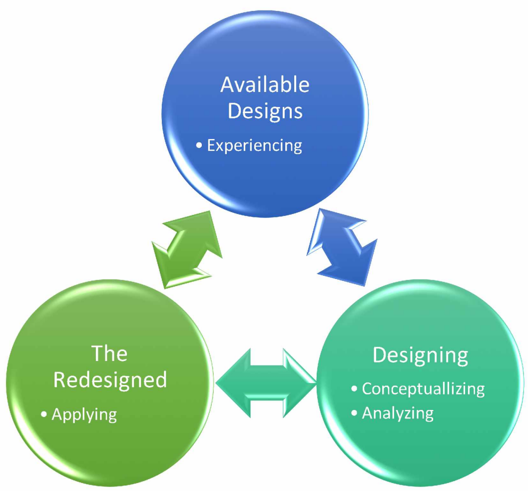


Figure 2.2: The design cycle

meaning maker creates is a new design, an expression of their voice which draws upon the unique mix of meaning-making resources, the codes and conventions they happen to have found in their contexts and cultures" (Cope & Kalantzis, 2009, p. 177). This is similar to their discussion of the cycle of the four thinking processes mentioned above—experiencing, conceptualizing, analyzing, and applying. Engaging in the available designs gives students opportunities to experience new information. The designing process is when students are conceptualizing and analyzing this new content and making decisions about how to use it. In the redesigned, students can now apply their new knowledge elsewhere.

Available designs are the input for meaning making. Available designs might include the facts in a book, the steps in a math problem, and the rules of grammar. Learners might also bring in available designs from previous knowledge: demonstrations they have watched; similar problems they have solved; or their related past experiences. This is what a student has available to them that may lead them to learn and make meaning. During this stage, students may ask themselves: What facts am I learning about the topic? How is this problem similar to problems I have solved before? What steps do I need to take in order to solve a problem? What happened that caused this to happen?

Designing is the analysis of meaning making. Students take what they learned and develop opinions about their topic. They think about what the information means to their worldview. This step is very important for students to be able to solve other problems that may have slightly different steps. They may put parts of available designs together in new ways. While there is typically one right answer in math problems, sometimes there are different paths to the solution. When solving equations, for example, sometimes the solver needs to add instead of subtract. Other problems require knowledge about key concepts such as a negative sign, so they can analyze if it is acting like a subtract operation or if the negative is combined with another operation. Potential questions for this stage of the cycle include questions like this: So, what does this mean to me? How do I feel about what happened in the story? Which process makes sense to solve this? How should I use this to solve my next problem?

Redesigning students' understanding of a topic requires them to think about what to do with the new information. It requires them to go from thinking about how the topic applies to them and start thinking about how it applies to others and the world around them. “The act of designing leaves the designer Redesigned. As the designer makes meanings, they exert their subjectivity in the representational process, and as these meanings are always new (‘insights’, ‘expressions’, ‘perspectives’), they remake themselves” (Cope & Kalantzis, 2009, p. 177). This step is very important in solving word problems or developing an understanding of the world of mathematics. The students remake themselves as they develop their understanding and their skills. In order to solve word problems, the solver needs to be able to apply the topic in order to break down how to solve it. They also use this knowledge to determine the best way to solve a problem. Sometimes, like on standardized tests, students are given math problems without having been told the process they should use to solve it. In these instances, students must have an understanding of mathematics that allows them to pick the appropriate methods to solve a problem. Because this design process is not linear, a student’s path through the cycle is never predictable. Potential questions for this stage of the cycle include: Now what do I do because of what I learned? Why does what I learned matter? How do I use this knowledge to keep growing as a learner?

This cycle leads a student through meaning making, but because of the nature of learning, there is not a set path for students to move through the stages in a linear way. When teaching, it is important to make sure students make it through all three portions of the meaning making cycle. We need to give them time to reflect on the design and redesign portions without spending all of our time on available designs. We could give them all the information in the world, but if they do not draw conclusions on it for themselves or others, they will not make meaning from it. Students learn the best when they are treated as active learners. The design cycle allows them to interact in the designing and the redesigning portions where they can be those active learners.

In a math class, it is very easy to learn formulas, processes, and vocabulary words short term. One way I incorporated this into my classroom was to have my students write about whether they preferred learning through teacher-directed instruction or *Khan Academy* (khanacademy.org). *Khan Academy* is an online video and question service that allows students to see a mathematical process through videos then apply that to similar problems. I had them write on a paper in groups of three and let their partners know which method they preferred and why it was best. While this did not analyze a mathematical process, it was a way for students to look at how they were learning and communicate to me how they felt about their learning.

They also are asked to use the design cycle when I give them slightly skewed problems on their classwork and tests. When I started out teaching, I was frustrated by my students' scores on chapter tests. I had taught them the material, they completed worksheets, and we reviewed as a class, so why did they fail tests so often? I started printing the test the day before and going over every problem with the students on the classroom white board. I wrote the problem down and asked students to walk me through the problems. I allowed them to take notes and write down the steps themselves. The next day, I allowed them to use their notes on the test. Some of them still failed. I had done everything except blatantly tell them to copy the problem from their notes. I started to lose hope. Then I tried something else. The notes that I did in class taught students the process that I needed them to learn. The majority of their classwork had a very similar layout and process to solve. Occasionally, the problems would make them stretch their understanding a little bit farther and apply one more math understanding in order to solve it. The reviews for their tests recovered all the information they would need to access for their test, but they hardly ever looked the same as the questions on the test. I wanted there to be a different format than what they were used to and questions that would ask them to go one step further or apply one other piece of their knowledge. Despite student complaints that classwork and tests were always harder than what I taught in class, their test scores went up, for the most part. We have better conversations in class about why we are allowed to do certain things or how to get the last step of the problem. Instead of simply asking them to recreate the same processes they had seen before, I decided to help them be "designers" by having classwork that challenged them and test problems that asked them to go further. When I forced my students into the design cycle, they did not enjoy it, initially. But they did start to think of themselves as problem solvers, and when they did not have math for a semester, they missed that challenge and the reward of solving a difficult problem.

Learners use "four resources" to break down math problems.

The "four resources model" of reading emphasizes the four roles of interacting with a text, or the four resources that students should be able to use in order to make sense of what they read. Freebody and Luke (1990) describe the four roles as code breaker, text participant, text user, and text analyst. *Code breakers* are able to break down parts of a text (big or small) in order to make sense of it. *Text participants* are able to make connections from the text and use it to answer questions. *Text users* are able to describe the purpose of the text and explain how the text relates to them. *Text analysts* are able to take meanings from the text and decide if they are valid. "Consider these four components as puzzle pieces, which interlock concurrently through all lessons to create a cohesive framework" (Casher & Stotler, 2015, p. 26). For teachers, the four resources model is a way for them to make sure they are not focusing on one role too much. All pieces are important for students to make meaning and teachers should use them all appropriately. “Any program of instruction in literacy, whether it be at kindergarten or in adult ESL classes or at any points in between, needs to confront these roles systematically, explicitly, and at all developmental points” (Freebody & Luke, 1990, p. 15). I would also argue that, since all subject area teachers should be language teachers, these four resources are important in every classroom.

In math, it is harder for people to think of how students use linguistic text (reading and writing) to make meaning. In history, science and English, textbooks are used frequently to help students learn and to help them make meaning, but in math, textbooks are seldom used, other than for solving the exercises in them. However, this does not mean that math students do not use language or texts. The following explanation is how this four resources model helps me think about how my students make sense of math problems. In math, code breakers are able to break down the parts of a question or a set of directions to figure out what exactly is being asked of them. They can use this to solve word problems or to answer multi-step questions. They can also break down complicated vocabulary words to figure out what they mean. Text participants are able to connect problems to themselves or to their past experiences. On a test, a text participant might remember a time before when they solved similar problems. They could also act out a word problem in order to conceptualize it and solve it. Text Users are able to explain the steps they use to solve a problem, use written steps to learn a new skill, and discover why a particular question needs a particular formula or process. Text analysts are able to take a solved problem and decide if it was done correctly or not. If a problem was solved incorrectly they can fix it (error analysis). All of these skills are very important for students to be able to call upon when solving a problem. In English class, these skills help students make meaning of a text, but in math, these skills help students become critical thinkers and problem solvers that do not need to be explicitly taught every type of problem.

I taught my geometry students common roots of words that they would see in their vocabulary. One time, on a test, a student asked me what a segment bisector was. I had him break the term into segment, bi-, and sect-. He knew that a segment is a part of a line, he knew that bi- means two, and he knew that sect- means break or cut. So, I asked him what he thought a segment bisector was, and he told me it was something that cuts a line in two equal parts. Without telling him what it means, I guided him through using his code breaker skills to understand the problem. Even though he was mostly using his code breaker skills, the other resources were important here too. “Rather than conceptualizing the four resources as a hierarchical set of competencies, the four practices are ‘nested’ within one another, influencing each other and blurring the distinctions between the various perspectives or practices described” (Serafini, 2012, p. 161). He was also a text participant when he thought about the possible meanings of the word. He was a text user when he took the two non-contextualized meanings and brought them together to mean something with a mathematics lens. Just like the design cycle and other theories of meaning making, there is not one linear path to think through new information.

Students of any subject need to be taught how to use texts and different sources of information in order to grow as a learner in that subject. “Literacy is a multifaceted set of social practices with a material technology, entailing code breaking, participation with the knowledge of the text, social uses of text, and analysis/critique of the text” (Freebody & Luke, 1990, p. 15). In order to help our students be active learners by breaking down the problems themselves, teachers should use the four resources model to ensure that they are guiding well rounded students.

Learners use multiple modalities to interact with math problems.

Cope and Kalantzis also explored multimodality as a part of their multiliteracies framework. When making meaning, a student can learn in many different modes. Many of the modes overlap and are used at the same time. Cope and Kalantzis (2009) define seven different modes of literacy that students can learn through: written, visual, oral, auditory, tactile, gestural, spatial:

- *Written:* Learning through written word is a popular form of learning that does not need much explanation. Reading books, writing letters, and reading subtitles in a foreign movie all deal with learning through written words.
- *Visual:* Visual learning is a key component of any math class. Students see how a problem is solved or how a formula is used so they can use that same method on their own. Visual learning is also key in many other concepts as well: learning how to cook by watching your parents, learning how to make a beaded necklace by watching an elder, and learning how to write the letters of an alphabet by looking at handwriting diagrams are all examples of visual learning.
- *Oral:* Oral learning is when the learners themselves are speaking in order to learn. This mode lends itself most directly to learning in a speech or debate class or in a language class. In order to learn language or how to debate, the learner must speak in that format in order to learn it. However, it can also be used in a math class as a student explains the steps to solve a problem, in a history class as a student explains the consequences of war

to a partner, or in any collaborative assignment as students explain their thoughts to another learner.

- *Auditory:* Usually when students are learning visually, they are learning through auditory methods as well. Most concepts that we are shown how to do are accompanied with a spoken explanation that describes what we are seeing happen. However, this can also happen on its own when we are listening to music, hearing a podcast, or have to follow a teacher's spoken directions.
- *Tactile:* Tactile learning is when students are able to physically touch whatever they are learning and experience the learning through their five senses. Science labs are a very tactile way of learning. Cutting a seal, driving a car, and playing a sport are all tactile ways of learning.
- *Gestural:* Dancing and other types of movement are ways to learn with gestures. When teachers come up with movements for vocabulary words and have students make that gesture when saying the word, they are using the gestural mode. Choir teachers may have hand signals for each step of an octave (do, re, mi, fa, so, la, ti, do) that students make with their hands while singing. Gestural modes of learning are usually paired with another mode in order to learn.
- *Spatial:* Spatial learning deals with the relevance of the learning topic to the student (is this about me? My family? My community? The world?), but also deals with if they are learning interpersonally, with others, or intrapersonally, within their own mind. It can also incorporate how items are laid out in a space.

Throughout the seven modes of literacy and learning, the different modes can overlap, and there are also modes that students are most comfortable in and prefer to learn in. It is a teacher's job to make sure that they have opportunities to learn in their comfort mode but also have the opportunity to work with other modes to strengthen their adaptability. Cope and Kalantzis (2009) argue that these different modes are not parallel—they do not exist one at a time. The modes overlap sometimes, but they are also separate at other times. Cope and Kalantzis argue that:

> The parallelism means that you can do a lot of the same things in one mode that you can do in the next, so a pedagogy which restricts learning to one artificially segregated mode will favor some types of learners over others. It also means that the starting point for meaning in one mode may be a way of extending one's representational repertoire by shifting from favored modes to less comfortable ones. (2009, p. 180)

Since the modes are not parallel, it becomes more crucial for teachers to use multiple modes so their students can experience all aspects of a topic. Cope and Kalantzis say that it would be like the comparison of a novel and its movie adaption. There are certainly overlaps, but each mode has its own nuance that adds more to the story. It would be like teaching math without being able to show the problem to the student (solely verbal), or showing the student the problem without ever explaining the process or why it is able to work (solely visual).

Although most math instruction depends on written, visual, and oral modes, a teacher can engage a wider range of modes. For example, in Pre-Algebra, when teaching about graphing ordered pairs, I put a coordinate plane on the classroom floor in tape. Students, in teams, had to pick an ordered pair, write it on their small white board (written or visual), communicate with each other on where that point was (oral and auditory), and one team member would stand on that point (tactile and spatial). While not required, many students gestured to each other how to go to the point by motioning to their team member. Additionally, we discussed which was the x-and y- axis by making sweeping motions with our arms. We turned the activity into a competition to spark the competitive nature that my students have. Previously I had the students practice graphing coordinates, we talked about the vocabulary involved, and we made foldables for our notebooks. While all of those helped us get to the point of being able to stand accurately and quickly on a point, I know that graphing did not click for many of my students until having that multimodal experience. When I asked them to interact with the material and communicate with each other, they were able to make meaning more efficiently.

Teachers Assist in Their Students' Meaning Making

My first assumption that learners are active meaning makers, and my second assumptions addresses the role of the teacher. My second assumption is that teachers (and the language they use) play a very important role in students' meaning making (Kalantzis & Cope, 2008; Duyke & Matusov, 2016; Shanahan & Shanahan, 2012; Rainey & Moje, 2012; Freire, 2005; and Morrone, 2004). As a math teacher, I have sometimes focused too much on math, forgetting to pay attention to the language and literacy tools that are so crucial to what I was teaching. In order to balance my lessons and ensure that my students think of themselves as mathematicians, I needed to ensure that my students had ample opportunities to be active learners instead of passive recipients of information. They typically will not engage with the material if they are simply copying what they see. No matter what students are learning, they need a balance of language and content so they can interact with what they are learning. This allows them to process their learning and think critically about their new learning, what their new learning means to them, and how to connect their new learning as they move forward.

The following discussion addresses three issues related to my role in supporting my students' meaning making. First, teachers in general can enhance active learning by helping students develop a sense of agency in their classroom. Also, math teachers should do this by incorporating and emphasizing the importance of mathematical discourse during instruction and independent work. Finally, in my classroom, I wanted to make sure I helped my students make meaning by developing an atmosphere where collaboration was encouraged; and I was a role model for my students in collaboration, perseverance, and discourse.

Teachers can use language to help students develop agency.

One way to encourage learners to take an active role in their learning, is to help students develop a sense of agency in the classroom. When I got frustrated with how often my students asked for help, I knew I wanted to change our classroom interactions. At first, I thought they just needed to be more perseverant, and that would develop naturally as they got to know me and got used to my teaching style. I did not think my words had a lot to do with this transformation.

When that did not happen naturally, I started to think about what I could do to help my students be more independent problem solvers. I knew that if I wanted them to be prepared for life after high school, I needed to help them become more independent. "If nothing else, children should leave school with a sense that if they act, and act strategically, they can accomplish their goals" (Johnston, 2003, p.29). I had a fresh goal in mind, but I had no ideas for how to accomplish it.

Johnston (2003) describes several questions (Table 2.1) that could be used to encourage certain qualities in students. One of those qualities is agency. Johnston explored different ways for teachers to work with their students to create a classroom environment where agency is expected and appreciated. All of Johnston's suggestions are questions that teachers ask their students; they are not things that students decide to do to create agency in themselves. When I thought that my students just needed to work harder in order to accomplish independence, I was not doing my part to guide them in that development. When teachers make an effort to develop agency and active learners in their classrooms, students will begin to learn the content more easily. Students will start to think of themselves as mathematicians, scientists, authors, and historians.

Some of these questions stood out to me as I started to think about how I mediate for my students and how I wanted them to talk about their mathematical problem solving; thus,

Table 2.1: *Questions to Develop Agency and Math Applications*

Questions to Develop Agency	**Applications in a Math Context**
"How did you figure that out?"	Asking students how they figured out their math problems allows students the opportunity to explain their thought process and think about the strategies they used to solve a problem.
"What problems did you come across today?"	Even though this question is not as applicable, asking students what they struggled with can help them identify what types of problems they need more practice on or why they are struggling with them.
"How are you planning to go about this?"	This question gives students the opportunity to explore their problem-solving strategies with intentionality.
"Which part are you sure about and which part are you not sure about?"	When asking students to break down what specifically they are struggling with, a common answer is, "everything!" This question helps students realize that their mathematical knowledge builds from previous knowledge, and they are not starting with nothing.
"Why…?"	Even though this question is quite broad and can be applied to many things, in math, it is very crucial. When students understand the "rules" of math and the logic behind the steps that they take, it becomes much easier to apply this knowledge to unfamiliar problems.

mediating for themselves. "When a child encounters a problem, asking, 'What can you do?' has several benefits. It reminds the student of her agency – 'I can do something' – and asks for an exploration of possibilities…. It requires the child to be in control of the exploration and selection of strategies, not just the exercise of them" (Johnston, 2003, p. 33). When my students ask me what to do next, if I had stopped to ask them, “Well, what *can* you do?” my students might have looked at me with blank faces, but they also might have started thinking about all the mathematical processes they did know. Since there are set “rules” for what you can do to a mathematical problem, asking students what rules they already know can help students see how much progress they have made in their mathematical knowledge.

This question also invites students to become inquirers--to start exploring different possibilities on their way to finding a solution. So often mathematical problems are treated as something you must solve correctly, the first time, with no mistakes, and you should know how to solve it as soon as you read the problem. This puts too much pressure on students who are unsure. Knowledge is fluid and constantly changing in a student’s mind. “And the key issue of language use [in learning] is agency and subjectivity—the way in which every act of language draws on disparate language resources and remakes the world into a form that it has never quite taken before” (Kalantzis & Cope, 2008, p. 204). If we want our students to be independent while they are persistently shifting their knowledge and connections, we need to make sure that what we say to them convinces them of their capabilities. In science, learning is often framed as experiments, and failures are often thought of as learning experiences and invitations to think up a new solution. Too often, in math, when students are wrong or if they have to attempt a problem over again, this makes them want to give up. If more teachers started asking *what can you do?* to help students see math as an inquiry, more students would develop resiliency when attempting to complete a math assignment.

“Why” questions are so crucial in a math class because the logic of math can be very difficult to understand, but is so necessary for meaning making. Johnston explains, “ ‘Why’ questions also develop children’s persuasion and argumentation abilities, and logical thinking . . .

Asking why children do or say the things they do helps them develop the consciousness and hence ownership of their choices" (2003, p. 37). When students have ownership of their choices and their learning, they are more likely to develop a sense of agency in their learning. Their ability to understand the world of mathematics gives them the tools they need to solve problems like a puzzle. This will help make math fun and challenging while increasing their motivation and perseverance when they do hit a snag. Johnston says:

> Developing in children a sense of agency is not an educational frill or some mushy-headed liberal idea. Children who doubt their competence set low goals and choose easy tasks, and they plan poorly. When they face difficulties, they become confused, lose concentration, and start telling themselves stories about their own incompetence. In the long run, they disengage, decrease effort, generate fewer ideas, and become passive and discouraged. Children with strong belief in their agency work harder, focus their attention better, are more interested in their studies, and are less likely to give up when they encounter difficulties. (2003, pp. 40-41)

Developing a student's agency is not just something that is a good idea for teachers. It is something that teachers need to do in order to mold problem solvers and meaning makers who will keep a passion for learning after school. Without motivation or belief in themselves, students will only complete assignments to get a grade or a degree. Without opportunities to analyze, problem solve, or argue for their beliefs, our students will lose their curiosity. They will not think of themselves as mathematicians, scientists, authors, or historians.

Math teachers use mathematical discourse to help students problem solve.

I want my students to understand math language better. By that, I mean I want them to be confident in their use of key vocabulary terms and I want them to know how to explain their thought processes when they have solved a problem. I want them to be able to break down word problems to figure out what they are asking for and I want them to understand the importance of directions so they will read them before immediately asking for help.

In my math class, I try whenever possible to allow students to reflect through language, but I am nowhere near where I want to be. I believe that including language in the content is so much more than having students read, write, listen, and speak in a class period. Students can write down a math problem without ever using English. They can tell a partner what answer they got without reflecting on how they got that answer. They can read a passage in a text without understanding the math or the language their eyes are gazing over. I have to be intentional about expecting my students to use language as they solve problems.

At the same time, teachers should not focus so inflexibly on language that they keep students from playing with the content. Duyke & Matusov (2016) says, “This mix of discourses, connections, personal meanings and authorial interpretive insights… are fruitful sources for students to build diverse understanding of a problem through playful exploration, mathematical modeling, the testing of diverse understandings and ideas, etc.” (p. 12). Teachers should encourage their students to use the discourse that they need to learn, but without the ability to interact, play, and experiment in the language, they will not be fluent in the language. For example, when learning about the distributive property, my students like to refer to it as the rainbow property. When we show that the term outside of the parenthesis multiplies to all terms inside the parenthesis, it looks like a rainbow. While “rainbow property” is never something they will see on a standardized test, this interaction with the language shows the students making connections and attempting to make sense of the language. I require my students to use the phrase “distributive property” because those words are important for their interactions in math, but I do not ban “rainbow property.” Teachers need to find a balance that requires students to use academic discourse and also encourages students to play with the language.

Disciplinary literacy, as described in Shanahan and Shanahan (2012) is not just teaching students how to read a text or look for key words to understand a passage. It is about teaching them how to read, write, think, and explain in each discipline as if they were experts in that field. “[The] foundational differences in the disciplines require differences in texts and language and therefore differences in approaches to reading and writing” (Shanahan & Shanahan, 2012, p. 12). In order to guide budding mathematicians, I need to teach them not only how to do math, but how to think like a mathematician. “Students would make greater progress in reading the texts of history, science, mathematics, and literature if instruction provided more explicit guidance that helped them to understand the specialized ways that literacy works in those disciplines” (Shanahan & Shanahan, 2012, p. 16). It means training history students to look at sources and know if they are credible or account for their side of the story; it means training science students to look at texts to discover connections; and it means looking at mathematics texts to learn a new concept (Rainey & Moje, 2012). Each discipline requires different skills and when teachers use disciplinary literacy in their classroom, it is their job to make sure students understand the language of their discipline and can communicate in it. I want my students to be able to communicate like mathematicians, so I need to teach them those specific skills. These communication skills would help them be better collaborators, active learners, and problem solvers.

Teachers must create an environment for collaboration in the classroom.

While I can do everything in my power to develop students’ agency and help them use the most advanced mathematical discourse available to them, my students will not make very much progress in math if they are not able to collaborate with one another. I have to create this atmosphere of collaboration in my classroom, so that my students feel comfortable asking me questions, asking their peers questions, and answering their peers' questions. The following discussion cites research that suggests that an environment that supports collaboration would include particular features, like teacher modeling, a positive affective climate, risk-taking, dialogue, and opportunities to work together.

Since my students do not like to make mistakes, I have to create for them an environment where it is okay to explore, work together, and learn from each other (with or without mistakes). "Children are extremely sensitive to teachers who do exactly the opposite of what they say. The saying, 'Do what I say, not what I do' is almost a vain attempt to remedy the contradiction and the incoherence . . . What is said has, at times, such a force in itself that it defends itself against the hypocrisy of one who while saying it does the opposite" (Freire, 2005, p. 98). I need to use the discourse and demonstrate collaboration for my students. If I only say "the rainbow property," and never use the technical term, that is all they will learn, and they will not be prepared to see "the distributive property" on a test. If I do not show them how to work together, they will not be prepared to collaborate with problem solving in the future. It can be scary for a student to bring up their own ideas in a group setting if they are not sure if they are correct, so it is up to the teacher to create an environment where it feels safe over time.

In order to encourage this risk taking, teachers have to work with their students in order to establish a positive affect—meaning they are generally happy, hopeful, generous, and resilient. The teacher has to intentionally create an atmosphere where the learners want others to succeed. In a study of instructional discourse, Morrone (2004) discovered that student motivation was directly linked to this type of atmosphere. In the study, a participant stated that, “a positive affective climate that promoted risk-taking was positively associated with students’ mastery orientation, help-seeking, and positive emotions associated with learning fractions” (p. 24). I need to use this positive affect to teach my students that math can also have experiments, failure, and problem solving through mistakes.

It can be difficult to know how to develop a classroom that lends itself to collaboration. It is equally difficult to know, once you have tried, if your efforts to create this environment have worked. “Speaking to and with the learners is an unpretentious but very positive way for democratic teachers to contribute to their school to the training of responsible and critical citizens, which we need so badly, and which is indispensable to the government of our democracy" (Freire, 2005, p.115). Freire has also called this “dialogic” learning. When students are comfortable collaborating, they will be comfortable enough to experiment with the dialogue of the intended discipline and vice versa.

Creating a space where students can collaborate without fear lends itself to the development of academic discourse, which, in turn, develops students’ agency in that discipline. Encouraging them to communicate with each other is encouraging them to learn from each other. “Monologic instruction alone is not sufficient. Not only do children not always understand what they are told and so need to engage in clarifying dialogue to reach the desired intersubjectivity, but, individually, they frequently have alternative perspectives on a topic that need to be brought into the arena of discussion for further exploration” (Wells, 2007, p. 263). This stance removes the teacher as the sole source of knowledge in the classroom and helps develop other students as new sources for help and guidance. This allows the students who see themselves as sources of knowledge to develop their perception of themselves as budding mathematicians.

When students collaborate in the classroom, they take risks, build knowledge, allow themselves to experiment, develop their agency, learn to use mathematical discourse, and they start to think of themselves as active learners who could be mathematicians if they wanted.

In my classroom, I encourage natural dialogue to happen. While I do put students into groups and have them solve problems together, I prefer to encourage the collaboration that centers on student choice, as we try to solve problems together. While taking notes, I allow my students who volunteer to take turns leading the class through a problem. This allows them to check their misconceptions, ask questions about what they are unsure of, and allows me to check in on the students who do not volunteer. It also allows these students the chance to see their peers becoming mathematicians who make mistakes. Sometimes we laugh at those mistakes, and more student-driven questions come up because those mistakes are brought up when I am no longer the one leading the class. I think this is what Freire means by "dialogic learning."

I also allow students to work together on their classwork. I tell them that they are not allowed to write something on their partner's paper and they are not allowed to blatantly copy off another student. If they are able to follow these guidelines, I am okay with students leading other students through a problem step-by-step. I am okay with students not being confident enough to work through a problem alone, *yet*. Allowing them to work together builds the expert partners confidence while allowing the novice partner to hear the math problems solved with an explanation that is different from mine. These partnerships happen naturally when my students set out to work on classwork and they are constantly changing. Even when the students in the group remain the same, who gets the role of novice or expert has never stayed constant throughout the class. This allows the partners to truly collaborate naturally—without me telling them how and when they should collaborate.

Teachers and Students Must Collaborate in the Classroom to Make Meaning

Many researchers suggest that, while the student's role and the teacher's role are both very critical. If they do not work together to make meaning, their work will not be as effective. To understand this kind of collaboration, I found two concepts particularly useful—one theoretical and one practical. First, sociocultural theorists focus on the role of mediation, which is explained below. Second, teachers have found that "think alouds" can help make meaning making "visible," and can, therefore, support collaborative problem solving. Think alouds are also explained below.

It is important, however, to first acknowledge that this collaboration between teachers and student is not easy. Sometimes, there are teachers that do not focus on listening to their students. These teachers often see themselves as the sole or main source of knowledge in the room and become increasingly authoritarian over time. Freire (2005) states:

> I can affirm that if teachers are constantly authoritarian, then they are always the initiators of talk, while the students are continually subjected to their discourse. They speak to, for, and about their learners. They talk from top to bottom, certain of their correctness and of the truth of what they say. And even when they talk with the learners, it is as if they are doing them a favor, underlining the importance and power of their own voices. This is not the way that democratic educators speak with learners, not even when speaking to them. Authoritarian educators are preoccupied with evaluating the students, with seeing whether they are following or not. (p. 114)

By keeping a focus on my students and hearing what they have to say about their learning of mathematics, I am able to help them develop their active learning skills instead of increasing their dependence on me. If I act as the main source of knowledge in my classroom, my students would never be independent from me, and, therefore, they would not be able to develop their agency.

Since teachers and students both play important roles in the meaning making process, the best progress happens when teachers and students are able to collaborate in that meaning making. For a long time, I thought that meant that I needed to make all of my abstract math concepts fit into culturally relevant, inquiry-based projects. I agonized when I could not come up with a way to effectively turn graphing into a "mapping the village" project. While I was developing my focus for this research, I discovered that I can have my students collaborate with their peers and with me in typical, abstract mathematics activities.

Teachers and students mediate to create meaning.

Teaching has started moving away from lecture structures and into collaboration and group problem solving. In math, this structure is often uncomfortable for teachers and students alike. Lectures followed by worksheets are a typical structure for a mathematics classroom, and most topics are easiest to directly teach to students. The switch to collaboration encourages teachers to eliminate notes and worksheets completely, but this could be detrimental to students' learning of abstract math concepts. Instead, teachers should work on how they mediate for students and with students in their curriculum (Thompson, 1991; Wells, 2007).

As explained above, mediation is the process of teachers and students working together to create meaning. One way to do this is by using mediational tools such as language (explaining a problem together), drawings, posters, and note templates (Lantolf et al., 2015). However, teachers need to be cautious when they are helping students or providing mediation. In a desire to support students, educators may inadvertently foster a reliance on that support rather than on independence and mastery of content. "'What is being practiced [by the student]' is normally in the mind of an adult observer. We *intend* something to be learned, and we have students 'practice' it. What they actually learn can be quite a different matter" (Thompson, 1991, p. 268). If I want my students to practice a concept, but instead, they practice asking for help and having me do the work, that is what they will learn. I have to be cautious with how I help my students so that I do not unintentionally cripple students' understanding. In an effort to help students, I need to make sure my mediation increases agency instead of learned helplessness by gradually releasing responsibility to my students.

One way to mediate for students is to enhance the relationship between teacher and student, so the student feels comfortable learning and working in the classroom. "As adult and child strive to understand and be understood, intersubjective agreement, when it is achieved, both strengthens their interpersonal relationship and enhances the semiotic resources that enable the child to act on the social and material world" (Wells, 2007, pp. 254-255). Both teacher and student learn from each other, especially when there is reflection, like in note templates or think alouds, which I will explain in more detail below. When the interpersonal relationship is strengthened, I am able to help my students better and understand them better. They will also feel more comfortable coming to me when they do not understand. Without trust in the relationship, students will not be open enough for the teacher to mediate effectively. Johnston (2003) claims:

> This experience of 'thinking together' or 'distributed thinking' is an example of what Mercer calls an 'intermental development zone' or IDZ – a more social framing of Vygotsjy's zone of proximal development (ZPD)…. The IDZ concept has an advantage over the usual interpretation of the ZPD in that the process is nonhierarchical….it is a

process in which mutual participation produces development without the associated asymmetrical positioning. (pp. 68-69)

When I use mediation with my students, sometimes I am helping them, sometimes they are helping their peer, and sometimes they are helping me. No one is perfect, and there will be times when I make mistakes in my classroom. How I handle myself in those mistakes can teach my students how they should handle theirs. I had a math teacher in high school, who did not handle mistakes well. While she was teaching at the board, if we pointed out an error to her, she would blame the marker for writing the wrong thing. It felt like she was telling us that mistakes were not acceptable and mathematicians do not make them. When my students correct me or we solve a problem, just to find out our whole process was wrong, I try to act graciously, thanking them for catching my error or being patient as we work through our corrections together. By trying to show them how mathematicians can make errors while working with other people, I am trying to show my students how to mediate their learning with me and with each other.

As well as teacher with student and student with student mediation, I have also discovered mediational tools that I can use in my classroom. By using the resources in my classroom to help my students learn, I am able to develop their independence even more (Jewitt, 2009). One mediational tool that I have really enjoyed using is note templates. These outlines can help students see more structure and language in their mathematics and can guide students when I am not there. By having structured notes as a resource when I am not there, students can see more than just worked examples. These "extras" could include written steps, helpful hints, things to watch out for, and vocabulary terms. These templates are serving as a mediational tool. Before, I was the sole mediator, but now, I have this tool to help me mediate my students' problem solving. These tools allow me and my students to problem solve together during the times when lecture and worksheets are more practical than hands on projects. No matter what I am using to increase mediation in the classroom, all additional resources have a chance to help my students, and I need to test out which ones are the most effective.

Teachers can implement the "think aloud strategy" to increase metacognition.

Since I wanted to do my research with the typical problem solving of a math classroom, I had to figure out a way to assess student learning other than just looking at what they were able to produce on a worksheet or quiz. My focus on language brought me to think about using the think aloud strategy to assess a student's problem solving. This strategy is exactly what it sounds like—students are asked to voice the thoughts they have so teachers can assess their thoughts as well as what they write down (Kucan & Beck, 1997). To me, this is more genuine than asking a student to write down their thoughts because the written language task often trips students up and changes their focus.

> "The purpose of think-aloud is to help second-language learners develop the ability to monitor their reading comprehension and employ strategies to facilitate understanding of the text . . . The open-ended nature of the Think-Aloud Strategy is a benefit for second-language learners because this nondirective approach requires readers to stop and explore the text." (McKeown & Gentilucci, 2007, p. 136)

While think alouds were originally used in ELA classrooms, I think they have a place in any learning setting.

In mathematics, specifically, think alouds are one way that teachers can start to see why their students make certain mistakes or how they think and problem solve correctly. "Using think alouds in a math classroom allows students to stop periodically, think about their thought process, and verbalize what is happening in their minds as they read and solve word problems"

(Bernadowski, 2016, p. 5). This process can be used for more than just word problems, though. Just asking students to verbalize their thought process on any problem can bring many insights to what our students are able to do. "By listening to our students as they solve problems we should be able to learn not only what they struggle with but also why. Teaching to these misconceptions rather than to the test may be a better way to help students navigate their way through developmental mathematics" (Secolsky, Judd, Magaram, Levy, Kossar, & Reese, 2016, p. 15). Think alouds help me, as their teacher, know exactly what misconceptions they have or what they understand really well. This strategy gives me insight that I would not get from looking at a solved problem on a worksheet.

For my research, I knew I wanted to focus on my role in the classroom and how my students are able to use language in their problem solving. The concept of mediation helped me think about how I wanted to support my students, and think alouds helped me realize how I could look into what my students were thinking and how they were using language. Both of these specific concepts were consistent with my broader conceptual framework (Figure 2.1), including my assumptions that students are active meaning makers, that teachers can and should assist in that meaning making, and that teachers and students should collaborate to solve math problems. Together, these concepts from the research literature helped me start looking into my research questions:

- How do my students talk about their math process?
- How do I mediate their problem solving?

Chapter 3: Research Methodology

In math class, it is very important for students to understand not only how to do certain procedures, but also how to critically read the questions they are asked, understand what procedure they need to use to solve the problem, and explain why those are the right procedures. I want my students to proudly answer how math discourse and justifications of their work affects their ability to solve math problems. A lot of this work needs critical literacy skills—something that is not often taught in a math class. My students are good at solving math problems when they know what procedure to use, but they do not always know why they are doing those procedures. Because of this, I want to look into student agency and my role in increasing their meaning making and collaboration.

Research Questions

In order to look at math discourse and how my students are learning, I have many questions. Throughout the past year while planning this research, my focus has changed significantly in relation to several issues. Last summer, I wanted to look at translanguaging and how it affects math language. Since then, I have dropped the translanguaging focus and have changed my focus to math discourse in general, rather than the linguistic issues. Additionally, at the beginning of my planning process, I wanted to look at standardized data to determine growth. Now I am more focused on the specific words students use, how they are able to walk someone through their thought process, and what my role is in this process. Table 3.1 traces those changes.

Table 3.1: *My Research Questions and How They Changed Over Time*

Month Year	How my questions changed over time
July 2017	What effect does translanguaging have on how my students use language to problem solve? What activities can I design to help my students problem solve with inquiry based projects?
Early November 2017	If I teach my students the necessary disciplinary literacy skills, how does it affect their standardized test scores? How does a focus on teaching math language affect students' math competence?
Late November 2017	What happens to students' problem-solving skills when they actively participate in the use of math language?
May 2018	What happens to students' problem-solving skills when they actively participate in the use of math discourse? How does students' math discourse change over time? Is there a correlation between standardized test growth and math discourse "growth"?
September 2018	How do students talk about their math processes? How do students use math discourse to talk about their problem solving? How do my interactions and language affect my students' participation and language?
My Focus Now	How do my students talk about their math process? How do I mediate their problem solving?

Study Design

For this study, I used teacher action research (TAR) for my research approach and constructivist grounded theory (CGT) for my analytical framework. Both processes had a big influence on my mindset as I went through the process of researching and analyzing my students and how they talk about their math process.

Teacher action research.

TAR is when teachers or principals perform research in their classrooms or schools, usually using mixed research methods, to take action and create positive change in the specific school environment that was studied (Mills 2018). TAR is appropriate for my research because whatever methods work for my students will not work for all students. My focus is creating a positive change in my classroom, and I hope that other educators will be inspired to create positive change in their school.

TAR is used by teachers as an inquiry cycle that allows them to investigate things that go on in their classroom. It helps them learn what works and what does not work by making observations in the classroom and analyzing them in systematic ways. Since there is always room to grow and learn in a classroom, TAR can happen over and over again. The teacher makes a hypothesis, observes, analyzes, and draws conclusions based on the analysis. Those conclusions could lead to more hypotheses, more research, and more experimentation in the classroom. In relation to the design cycle (Cope & Kalantzis, 2009), TAR is also a inquiry cycle that follows the same logic (see Figure 3.1). The research itself is the Available Designs; the findings are the Designing; and the Redesigned is the implications for future practice and new questions to prompt more research.

I believe this research is important for any math teacher no matter where they are because all students struggle with word problems and justifying their thought processes. When I started this research, I wanted to figure out my role in helping students make more sense of math problems. The outcomes that help my students might not work for other students, but the focus of language and looking into the importance of disciplinary literacy could spark the interest of another teacher to think about their classroom.

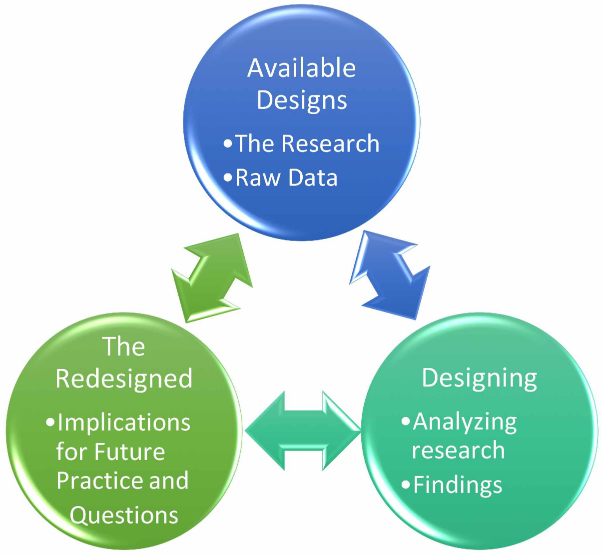


Figure 3.1: The design cycle as it relates to TAR

I believe my questions are important for the math community and teachers as a whole because our society is slowly becoming complacent. We are not thinking for ourselves as much since the answer to any question is just a Google search away. We are less likely to value hard work and doing something for ourselves since hiring freelance workers has gotten easier and easier. I believe we need to bring back critical thought and learning for the sake of learning if we want to progress in society. Being able to explain how we solve problems and what we do when we get stumped is not just a math skill; it is a life skill. If I can find ways to teach my students to speak the language of math, I think it will greatly affect how competent they are in math.

When I do research in my classroom, it is very important to me that it is meaningful for me and for my students. In order to create this positive change in my school, I need to figure out what works for my school without stepping on any toes or interpreting the data to fit my ideas. According to Mills (2018), teacher action research is trustworthy if it has credibility, transferability, dependability, and confirmability as defined in Table 3.2 below:

Table 3.2: *Characteristics of TAR Needed to Make Research Trustworthy (Mills, 2018, p. 156)*

Characteristics of trustworthiness	How I incorporated trustworthiness in my TAR
Credibility: For research to be credible, the researcher has to be able to explain any data that deviates from the norm. Not all data points will fit the way we want to, but if there is a large outlier, the researcher should be able to argue why it is an outlier.	If something does not work well for my students, there is no reason to lie or manipulate and say it does – that will not help my students learn. When I look for results in my research, I do not manipulate the results to show what I predicted.
Transferability: For research to be transferable, the researcher needs to understand that they are not trying to find truth statements that will fit all contexts and all students. They need to be able to generalize their findings in order to say something that applies to all educators.	I understand that my students are uniquely mine and any other classroom could need different things from an activity. I will do my best to make suggestions for activities or lesson components that can be changed to fit similar classrooms. When I make claims, I will narrate what happened in my classroom instead of saying that certain things work.
Dependability: For research to be dependable, the research needs to produce similar results no matter how often something is tested. If an activity raises test score significantly for one setting, it should also raise test scores another time with a similar setting.	My conclusions will come from patterns, not singular instances. I also will not make claims that certain activities are guaranteed to raise test scores.
Confirmability: For research to be confirmable, all researchers should be able to draw the same conclusion from the data. Statistics should not be manipulated to tell the reader whatever the researcher wants.	I will make sure that all data points I collect are in the students' best interests and that no one is left out simply for not fitting other data points. I will use multiple data sources, and debrief with colleagues to see if my inferences are reasonable. My long-term work with the students means I am already familiar with them as learners and will not be judging them solely on a few weeks of observations.

If TAR is not done with the students at the forefront, then there is no purpose in doing it. Being able to use TAR in my classroom was a huge eye-opener for me because it made me challenge my thoughts about myself as a teacher and how I guide my class. Because I looked at my data with an open mind and cared about my research being trustworthy, I was able to think through what worked and what did not work in my classroom. TAR was a great method to guide my thoughts and ensure that I was focused on my students and enhancing their math education.

Constructivist grounded theory.

For this study, teacher action research is my research approach. It guides my process and thoughts during the research process. Constructivist grounded theory is the analytical framework that I use within my TAR to think about my data and develop patterns and conclusions.

Definition.

CGT is a flexible, cyclic process that directs the researcher as they move through the process of analyzing and interpreting their data to develop a theoretical understanding of their subject (Charmaz, 2014, p. 1). It gives the researcher a systematic way of combing through their data, and it keeps the researcher focused on their data to find accurate patterns by using coding, memo-writing, and sampling with comparative methods. CGT is like a river that picks up rocks (or data points), swirls them around (analyzes them), and deposits them further along (either turns them into a category or sets them aside). Charmaz (2014) defines constructivist grounded theory as:

> Grounded theory methods consist of systematic, yet flexible guidelines for collecting and analyzing qualitative data to construct theories from the data themselves... Grounded Theory begins with inductive data, invokes iterative strategies of going back and forth between data and analysis, uses comparative methods, and keeps you interacting and involved with your data and emerging analysis. (p.1)

When researchers are combing through qualitative data, it is helpful to have a systematic way of analyzing it to help ensure credibility. At the same time, their readers need to be critical of the interpretations made as they decide if this information will inform their practices. CGT recognizes that there are many possible outcomes in a researcher's analysis and attempts to limit the range of different interpretations through specific guidelines in the analytic process.

Process for researchers.

While, CGT can be explained in a few statements, Charmaz (2014, p.15) expands the process into nine steps, seven of which are relevant to my study:

Table 3.3: *Adaptation of Steps of CGT and how They Applied to my Research*

Steps of CGT according to Charmaz	How I used the steps in my TAR
1. Collect and analyze data repeatedly.	I collected data in the Spring of 2018, decided it was not enough, and collected again in the Fall of 2018.
2. Analyze what the participants do and say – do not look for patterns yet.	Before looking for patterns, I transcribed the videos and took notes on what my students and I were saying.
3. Compare what can be noticed in the actions and words.	I looked for instances where the verbal steps they listed did not match the written steps they performed.
4. Look for patterns in the actions and words.	Every pattern was based on transcriptions and student work.
5. Put patterns into categories while continuing to systematically refer to the original data.	Every time I wanted to talk about a finding, I referred back to the transcription for my students' words and written work.
6. Develop theories based on patterns rather than describing existing theories or explaining how existing theories were used.	In my Chapter 5, I discuss my findings in Chapter 4. I went with what I thought was a logical next step as a teacher, based first on my data. I used current theories to affirm or elaborate on the findings grounded primarily in my data.
7. Look for data points that do not fit the patterns or categories.	If there was an instance that did not fit my pattern, I wrote about it, too.

Without a systematic process for analyzing data, an analyst can make data show whatever they want it to. CGT keeps the focus on the original data while giving enough flexibility to work for many different studies.

Connection to my TAR.

Constructivist grounded theory is important to my research because I want to look for ways to inform the mathematics education community and encourage critical thinking and problem solving. Without a framework that keeps me focused and trustworthy, my research loses credibility. CGT also guides my analysis by giving me specific processes to follow while looking at my data.

When a teacher researcher records an event in their classroom, there are many ways to take that recording and turn it into patterns. In Table 3.3, I briefly explain how my TAR did that. In my TAR, I transcribed my recordings (wrote down who said what), then coded them. "Coding is the pivotal link between collecting data and developing an emerging theory to explain these data. Through coding, you define what is happening in the data and begin to grapple with what it means." (Charmaz, 2014, p.113). Before coding, it was hard for me to even try to make sense of my data. The systematic process of coding and looking for patterns in those codes and the original transcription made the process of looking for patterns more focused and logical. In order to code my data, I made notes about what I saw in the transcriptions, thought about the math language my students used, looked at how I prompted or responded to my students, and looked at the math that my students completed.

After I coded my data, I was able to look at the codes and look at the transcriptions and student work to find aspects that were interesting. Once I pulled the interesting moments out, it was easy to find patterns and connections between them. When I tried to look for patterns before I had analyzed the transcriptions enough, I got frustrated because nothing was jumping out at me. I wanted to know how I would organize my data and the patterns I found from it before I had taken the time to code and compare within the data. Once I let go of the need to organize first and followed the steps more accurately, the patterns came together on their own. The organization of my analysis also changed after I followed the correct steps because I was able to organize it in a way that was more true to my data. It was very natural for me to want to force the analysis to happen, rather than letting it develop on its own, but the process did not flow until I took the time with each step.

Setting

The moderately sized village of Toksook Bay, Alaska, sits in the south-west corner of Alaska on Nelson Island. It is southwest of the larger cities of Anchorage and Bethel and sits nestled in the small Kangirlvar Bay. There are 135 houses ("Interactive population map," 2010), two general stores, a personally owned store, a post office, the Headstart center, a multi-cultural building (where bingo is played, presidents are voted for, and dances are held), a tribal council building, and even a small "jail" (a holding room that does not lock).

Our school, Nelson Island School, used to be split into two buildings; the elementary grades had one building while the middle and high schools had another. In 2008, they put an addition onto the middle and high school so we now have one school that educates K-12. Most of the elementary teachers got brand new classrooms with lots of space, doors that lock, and enormous windows that fill up with snow in the winter. Meanwhile, the secondary students and teachers are in large open spaces where walls are sometimes created from shelves, doors do not exist, and windows are coveted. Sound carries a lot and resources are pushed aside as the school library and workout room become makeshift classrooms. Teachers deal with critical management problems such as unlockable balconies and storage spaces (there is no lock on archery bows and arrows), staircases that students slide down, and disappearing students that slip out of chaotic, noisy classrooms with no doors.

Participants

I did research in two different phases, one in the Spring of 2018 and one in the Fall of 2018. After I recorded in the Spring and started to analyze those data in the Summer, I decided I did not have sufficient data and I wanted to deepen my research questions. This led to me doing another cycle of research in the Fall. The following analysis and findings focus on the Fall phase.

My Algebra class had 15 students, all of whom are at least half Alaska Native. The students that participated in my research were students in my Algebra class who wanted to participate and whose parents signed consent forms. Four of the 15 were involved in my study. One student was not given the opportunity to participate because he missed more than 50 classes in the semester. I did not talk to him about the research because he was not at school when we were doing assent and consent forms. One student said he wanted to participate, but when it came time to record him, he would not talk and his hand was shaking (I quietly told him that I did not need to record him if it made him uncomfortable). Nine students said they did not want to participate. All four of the students who said they wanted to participate said they did not want me to video record them, but gave permission for audio recording and work samples.

To ask my students about participating, I handed out the assent forms to the class as a large group and read through it with them. I paused every once in a while, to explain certain sections or answer any questions. At the end, I summarized and explained what it would mean to them if they were a part of the study. I went around and asked them the yes and no questions (for example, “Is it okay if I use your work?” or “Is it okay if I video record you?). As students responded, they seemed interested to know how their peers were responding.

Some students immediately circled "no" without hesitating. Other students faltered when I asked them the questions. I reassured them that the choice was up to them and that it would not hurt my feelings if they said no. I also told them that if they said yes, all they were agreeing to was letting me look at what they said and trying to find patterns. There was no extra work involved. Picking pseudonyms turned into a bigger process than I expected. Most of them wanted to use their actual name and others wanted to use inappropriate names. I chose pseudonyms for the students who said they did not care. Once I had my four participants, I was able to start the recording part of the research process.

Instructional Process

I teach high school math in a kindergarten through 12th grade school. Even though I teach all high school math classes at my school (Algebra I, Geometry, Algebra II, Pre-Algebra, etc.), for this study, I focused on my Algebra I class.

I think of my instruction as either a lecture day, a stations day, or make-up days. On a lecture day, I typically start class by handing my students a notes template and letting them look through the types of problems we will solve that day. The notes template includes fill in the blank information, written steps, and example problems to solve as a class. Then, I lead the class through the example problems and fill in the blank notes, checking their comfort level, and letting them work on their own problems when they can lead me through an example problem without my help. While working on individual problems, they can work with a partner and ask me questions to get help. I try to give them as little help as possible if their struggle seems to be productive. I also tend to direct them to their notes on the template if they ask general questions like, "How do I do this?" Often times, if their paper is blank, I will tell them to try *something* and if they are wrong or still stuck, then I will help them. When they finish their individual problems, I have students work on an online program called *Aleks (*aleks.com). *Aleks* is a math resource that allows students to work on math topics that the instructor can set up and gives the teacher feedback on what topics they are doing and how well they do on them. I had it set up for my students to work on Algebra I questions in their *Aleks* accounts. Students work at their own pace, so while some students were working on topics that I had not covered in class yet, other students were still working on the basics from Chapter 1.

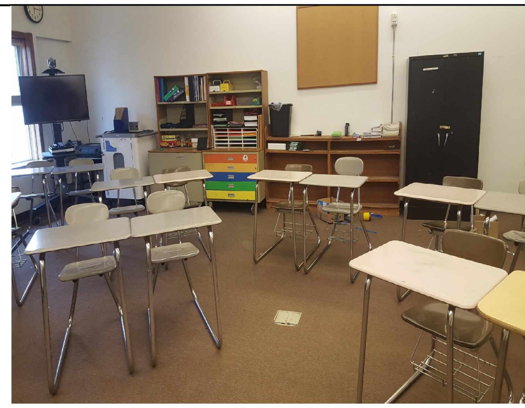
On a stations day, I had the desks set up in groups when the students come into the classroom. I either had the groups they needed to get in written on the white board, I verbally told them what groups to get into, or I would let them pick what they wanted to do first. What the students do in the stations depends on what we are working on and how many students are present, but typically, one station would give them time to work on Aleks, one station would give them time to work on a worksheet or assignment, and one station would have them take notes with me. The smaller groups allowed me to focus on students more individually as we were taking notes, and required more students to speak up and help the group solve the problems.

Classroom displays.

In my classroom, I do not like seeing white space, so there are a lot of decorations, furniture, and materials in my classroom.

Table 3.4: *Furniture in the Classroom*

Description	Pictures
• 16 desks • Table for group work • Teacher desk • Shelves and cabinets for supplies • Prize cabinet • "Turn-in" box for student work • Window • SMART board • Two white boards	Desk Arrangement, some shelves, prize cabinet, and computer cart

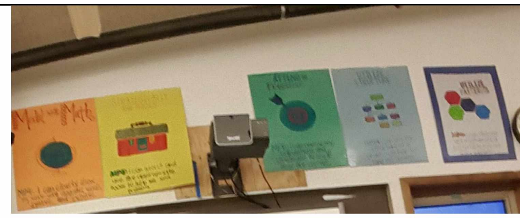
● Two bulletin boards ● Chalk wall ● Computer Cart	

There were 16 student desks with chairs attached set up in partners, angled towards the front whiteboard, and in two rows. Other typical classroom furniture was spread out around the room. My desk was at the back corner of the room, close to the doorway. While I was happy to have a window and a large amount of wall space, my classroom did not have a working SMART board or projector and there was no door to block out student visitors or distractions from the hallway.

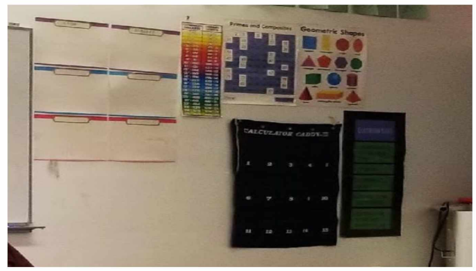
Table 3.5: *Wall Supplies and Decorations*

Description	Pictures

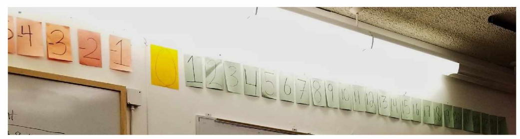
● File folders for extra papers
● Alaska display
● Student artwork and projects
● Posters with mathematical standards
● Word Wall
● Posters of:
    ○ Classroom Rules
    ○ Shapes
    ○ Prime/ composite numbers
    ○ Squares/ square roots
    ○ Daily objectives
● Calculators
● Student-made number line
● Daily schedules
● NIS posters
	Alaska Display:
	
	Some of the mathematical standards:



Posters and Calculators:



Number line



On the walls, there are file folders with extra assignment papers; an Alaska display; student artwork and past projects; posters with the mathematical standards; a word wall for each subject that I had that semester; informational posters on classroom rules, shapes, prime/composite numbers, squares and square roots, and daily objectives; calculators hung in pockets, a student created number line, daily schedules listing what times to switch classes, and Nelson Island School rule posters.

Table 3.6: *Classroom Materials*

Descriptions	
• Glue	• Construction paper
• Scissors	• Mini white boards
• Rulers	• Expo markers
• Protractors	• Textbooks
• Meter sticks	• Resource books for me
• Markers	• Art supplies
• Colored pencils	• Games (board/card/math)

● Graph paper ● Lined paper	● Other small supplies that I use

Materials on the shelves and in drawers include glue, scissors, rulers, protractors, meter sticks, markers, colored pencils, graph paper, lined paper, construction paper, mini white boards, expo markers, textbooks, resource books for me, art supplies, assorted games (board, card, and math) and other small supplies that I use.

Table 3.7: *Bulletin Boards*

Descriptions	Pictures
● I'm done, now what??? ● Vocabul-oggle	Vocabul-oggle 

One bulletin board has an "I'm Done, Now What?" display that gives students options for when they are "done" with their work. Most students will rush through assignments to try and have free time, so to discourage that, I put together options for what they can do. Free time is not one of them. Options include working on missing work, helping another student, playing a math game, doing the puzzle on the other bulletin board, working on Aleks, etc. The other bulletin board has a vocabul-oggle (Vocabulary Boggle). To solve this puzzle board, students have to find math words by tracing letters that are touching (up, down, left, right, or diagonal). For each word they find, they have to give a definition in their own words. Vocabul-oggle is the only form of extra-credit that I allow my students to do. For any other opportunities to raise their grades, I tell them to turn in missing work or re-take tests.

Research Procedures

As I explained above, I conducted two phases of research, one in the Spring of 2018 and one in the Fall of 2018. Brief explanations of both are included here. More details about the second phase are included in Chapter 4.

Phase 1.

Phase 1 occurred in the Spring of 2018. At the time, my research question was: What happens to students' problem-solving skills when they actively participate in the use of math language? This phase was not inherently different from my main phase of research, but through it, I was able to discover that I wanted to attempt the research again. During this phase, I was interested in really focusing on the specific math discourse that my students used to explain their problem solving. I used think alouds, but I also used written conversations and other methods to try to get the students to use mathematical discourse. During this phase, while I was transcribing the think alouds, I noticed that I was "helping" my students more than they asked me to. This was the first time I noticed that I may be a part of my students' lack of agency. Since I noticed this after the research phase was over, I decided to do another round of research, this time, with more of a focus on my input.

Table 3.8: *Research Phase 1 Participants*

Pseudonym	Age	Gender	Years in High School	
Damian		Male	1	
Abby		Female	2	
Adrienne		Female	2	
Angllu		Female	1	
Emma		Female	2	

Phase 2.

For Phase 2, I did four weeks of data collection in the Fall of 2018 after deciding that Phase 1 was not sufficient for what I really wanted to look at. Each week was designed to follow the pattern shown in Figure 3.1 and focused on a different form of teacher input:

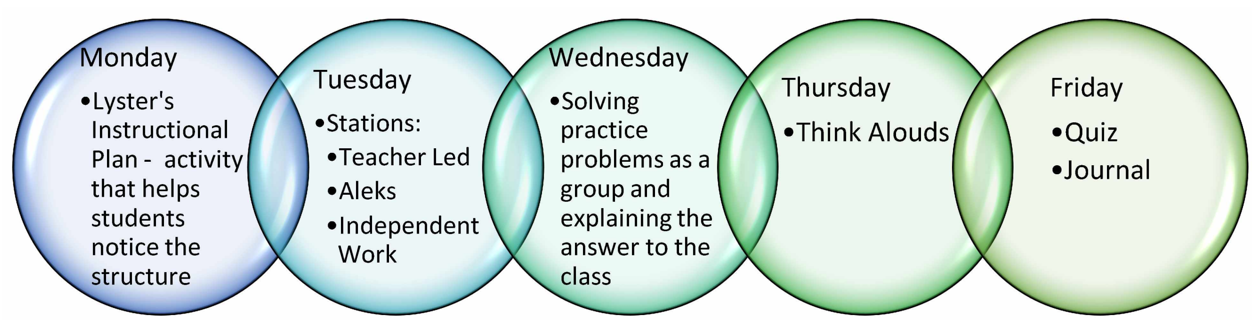


Figure 3.2: Planned phase two weekly structure

Each week I wanted to focus on providing a different amount of teacher input, but I struggled to do this because I ended up giving the amount of feedback that feels natural to me. I wanted to look at how my students did if I gave them no feedback, only asked them questions as feedback, or only answered their specific questions as feedback. I ended up not being able to control my feedback in this way. It was more natural for me to give feedback as a mixture of all three things: answering their questions when they asked; asking them questions if they started to veer off course; and sometimes saying nothing—waiting for them to self-correct.

Phase 2 had five participants with varying levels of participation. One participant was too nervous to be recorded and was not able to complete any transcription. Another participant struggled with self-confidence and did not want to be recorded anymore after his first video. Neither of these students asked to no longer be a participant; they just did not want to be recorded again. The other three participants wanted to participate throughout, but were not always present. We covered solving equations, solving inequalities, converting from standard form to slope intercept form (solving for y), and graphing the equation of a line throughout the four weeks.

Table 3.9: *Research Phase 2 Participation*

Participant	Equations	Inequalities	Solving for "y"	Graphing
Nathan	X			
AJ	X	X		
Ang	X	X	X	X
Apa-Kua	X	X	X	X

Each week I gave the students several opportunities to learn the concept through note taking, partner work, group work, and some discovery activities. I then recorded them individually solving a problem for that math concept on my iPad using an app called "Show Me." The app had the capability to record sound and what was being written in the app, thus allowing the speaker to show the viewer how to do something. I asked the students to tell me what they were doing while they did it. Once the student had finished the problem and explained their steps, I moved on to the next student until each student in the class had solved a problem with me. I only recorded the students that were participating in the study.

These videos of the students' work and the transcripts I made from the audio became my data sources for the analysis. In addition, a main data source was my Teacher Research Journal, where I took notes during the cycle and wrote research memos as I analyzed the data.

Data analysis.

After I got all of the recordings that I needed, I started to transcribe the videos. I put these transcriptions into tables, one for each math concept or video. Each table had a column for coding, the transcription line number, the transcription itself, teacher notes and observations, and pictures of student work. I would have included body language or other non-verbal things happening in the room, but since I only had audio recordings, this was not possible. These tables became my data ensembles (see example in Figure 3.2 below) for each video and described or showed everything that was happening in the video. At the end of each ensemble, I added a screenshot of their final product and notes on what I was observing at the time.

Apa-Kua and Ms. Boyd

The original problem: $-6x + 4y = 20$

Coding	line	Transcription	Notes	Pictures
Explaining the first step to solve for y	**1**	A: Minus Six x um plus six x	self correcting ("um" is used how I would say "I mean")	$\begin{array}{r} -6x + 4y = 20 \\ +6x \quad +6x \\ \hline \end{array}$
Confirming	**2**	T: [mhmm]		
Explaining the second step to solve for y (almost correct)	**3**	A: Okay, cross these off. Like like this? Uh, x? Or wait! Four y equals x plus twenty?	Needs the 6 with the x	$\begin{array}{r} -6x + 4y = 20 \\ +6x \quad +6x \\ \hline 4y = x \end{array}$
Disconfirming then confirming (the 4y)	**4**	T: [mm-nn] [mhmm]		

Figure 3.3: Screenshot of an excerpt from an ensemble

Coding was the hardest part of the analysis for me because I wanted a set number of things to say in the codes. I wanted to know my patterns before I coded so that I could just go through and code where I saw these patterns. I had to open my mind back up and allow the patterns to naturally develop after coding and writing a lot of memos. Once I focused on what the students were doing, one line of transcription at a time, and just coded what was happening, the patterns developed on their own.

After coding, I wrote reflections on each ensemble. I noticed that I felt like I was repeating myself throughout the different memos. Originally, I thought that my analysis would be analyzed into patterns by individual student and I would write a reflection on each student, but seeing all the repetitions across students and across math topics, I realized it was even more important for me to let the patterns develop on their own without even having a structure in mind. When I thought I was repeating myself too much in the different reflections, those became my patterns and eventually, my findings in my analysis.

In Chapter 4, I present the result of this analysis as my findings, using excerpts from the data as evidence. Some patterns had several instances in the analysis that fit, and some were simply interesting events that happened one time with one student. All of the patterns emerged from the data that I found interesting or noteworthy because of their relevance to my research questions, but does not encompass all of the intricacies of the four cycles or my algebra classroom.

Chapter 4: Data Analysis and Findings

When I first started researching how students explain their math process in the Spring of 2018, I did not have a lot of parameters. Over the summer, while analyzing these data, I realized I wanted more structure in my cycles and I wanted to be more intentional with my words so I could analyze them as well as the students' language. This led me to a second cycle of research in the Fall of 2018 in which I asked the students of my Algebra I class if I could record their voices and their work as they explained different math problems. Four students agreed to be in the study, but not all of the four were recorded in each type of Algebra problem (cycle). While they solved different types of math problems on an iPad, I had the iPad record their voices and anything they wrote down. To analyze the data, I transcribed all of the recordings, took screen shots of their work, and started to make notes and code using CGT (constructivist grounded theory, Charmaz, 2014). I then took what I found to be interesting and thought about what that meant for my students, myself, and my future practice. While I was thinking about the implications, I tried to keep in mind my research questions: How do students talk about their math process? How do I mediate their problem solving?

Description of My Students

In Toksook Bay, Alaska, there is a very large percentage of Alaska Native residents. All of the students in my Algebra I class in the Fall of 2018 were at least half Alaska Native. The school in the village is a K-12 school that was built in the 80's and in 2007 added an elementary wing. The high school and middle school classrooms were designed with an open concept (no walls), but since have seen walls with open doorways put up. The village is fiercely proud of their native culture, language, and values. The school boasts having the highest Yugtun (the language of the Yup'ik people) Proficiency Percentage in the district. Our school's Native

Yuraq Dance Team has traveled to Anchorage and Fairbanks several times to participate in the state-wide Alaska Federation of Natives (AFN) dance festival. Any student in Toksook Bay would tell you “[basket]ball is life” and the “play deck” right outside of the school is full from sun up to sun down in the warmer months with kids shooting a ball around or playing self-organized scrimmage games. Like any typical U.S. community, Toksook Bay has its problems with drugs, alcohol, and suicide. These issues can be overwhelming for our students, but they try their best to focus on their culture and the good things that can come from it.

The 16 students in my Fall 2018 Algebra I class were a challenging group for several reasons. A lot of these students had taken the class before and failed. Others had taken Pre-Algebra the previous year and passed, but did not have a lot of confidence in their math abilities. This was true for the four students that participated in my research. All of them had been told at some point in their lives, directly or indirectly, that they were not great at math. All of them had classmates who were put in a more difficult math class because of previous scores and grades. My research participants’ friends were in a "higher" math class, and they felt left behind or embarrassed by this “lower” status. I often got frustrated by this particular group of students’ lack of confidence and unwillingness to try a math problem independently. I believe this lack of confidence with math led to my students’ reluctance to participate in my research study.

When I introduced my project, I explained very carefully that it would not lead to any extra work; they would not have to do anything special; and that they would hardly notice the difference if they were in it or not. The only difference would be I either push a button when it was their turn or not. I also explained that their name would not be associated with anything I did. Only I would know it was them and only I would see the recordings. Even with all of my assurances, I had only four students of 16 say that they wanted to participate, and all four of them said I could not video record them—only audio.

Ang is the only girl who participated in my study. She was 16 years old and in 10th grade at the time of the study. She loves to play basketball and is on the Native Yuraq Dance Team. Her primary language is Yugtun, but she is comfortable speaking to her classmates in either language. She has attended Nelson Island School since I arrived in the village. She lives with her parents and four brothers, but also has an older sister who has moved out of the village. In school, she is often more than willing to do her work and is eager to learn; but she is quiet and will sometime be weighed down by emotional distractions.

Nathan is one of the boys who participated in my study. He was 16 years old and in tenth grade at the time of the study. He loves basketball and plays every chance he gets. His basketball passion has given him great teamwork skills. He is always willing to help out his peers and is very quick to ask for help from a peer or a teacher when he thinks he needs it. His primary language is Yugtun and is often heard speaking Yugtun when talking to his friends or Yup'ik adults. His parents are not very comfortable with English, so he speaks a lot of Yugtun at home. He has attended Nelson Island School since I arrived in the village. His parents adopted him as a baby, but by blood are his grandparents. He has one sister and one brother that were also adopted this way and grew up with him. In school, he knows he often struggles with the material, but that makes him fight that much harder to make sure his grades stay up. Anytime he feels himself falling behind, he will come in after school every day, as long as it takes, to get caught up again. He fights for every point he can get and will not let any missing assignments go by. His parents are elders in the community and often talk about keeping up in school never letting your motivation falter. I believe his parents and motivation to stay eligible for basketball keep this drive in him to always try his hardest.

Apa Kua is one of the boys who participated in my study. He was 15 years old and in 10th grade at the time of the study. He tends to have more of a “helper” personality. He is involved in more of the non-athletic afterschool activities. His family is very religious, and family is important to him. He is always willing to shovel a porch, help out with suicide-awareness events, play a musical instrument, or cook someone a meal. His culture is very important to him, and he will often quiz me on Yugtun words or ask me to participate in a community event. He is involved in both of cultural activities at the school—the native Yuraq dance team and NYO (Native Youth Olympics). His primary language is Yugtun and is often heard speaking Yugtun when talking to his friends or Yup’ik adults. His lives with his mom and his grandparents, and they often speak Yugtun at home. Sometimes when he is speaking English to me he will struggle to translate from Yugtun in his mind and will get tongue tied. He has attended Nelson Island School since I arrived in the village. His mom has one other daughter, but she has not lived in this village since I have. He is very close to his extended family and considers his first cousins to be his siblings. In school, he is well known as a “good kid” who is always willing to help out. Because he is always looking for someone to help, it is hard for him to stay caught up on his own work. For the majority of the class, his grades were borderline because he passed the quizzes and tests, but did not turn in a single assignment. During class, he was always willing to call out answers to help the class along during notes, but when it came time to do the practice classwork, he helped others finish problems, or he wandered around the room.

AJ is the last participant and the third boy who participated in my study. He was 16 years old and in tenth grade at the time of the study. He has a combination of an athletic and a social personality. He loves to play music, spend time with his younger siblings, and make videos with his dad, but he also is a very motivated member of the basketball team. His primary language is Yugtun but is very comfortable in either language. He lives with his parents, three brothers, and two sisters. He has attended Nelson Island School since I arrived in the village. In school, AJ likes to joke around, but he is always willing to help out a peer when they are struggling with an assignment. Throughout the basketball season, he constantly talks to his teammates and encourages them to try hard and get work done. He would walk them through the steps of math problems and answer any questions he could.

Description of Instructional Cycles and Analysis Process

Throughout the research that I did in Fall 2018, I recorded and analyzed students as they worked through different types of Algebra problems. I worked with my Algebra I class to cover many different aspects of the Algebra I curriculum. Our classes were on a block schedule, which means that it lasts for 100 minutes. Typically block classes only happen every other day in order to account for the longer times, but our block classes happen every day, so our usual yearlong class is reduced down to only a semester long. Because of this, we have half the number of days to get through the curriculum (although more minutes each day). This particular class was in the middle of the day—35 minutes of the class before lunch, lunch time, and then the remainder of the class after the lunch break. A classroom aide came into the portion of the class that was after lunch to help me with different tasks or to help a student with special needs.

My classroom layout is long, but narrow, so I have the whiteboard on the long side of the classroom allowing for desks to be spread wider with less rows. The students typically came in, sit where they want, get notes or the work for the day, and start looking at it or preparing their notebooks. Our math notebooks were composition notebooks that we glued the notes templates into. The students knew when I handed them notes to get out their notebooks, glue the papers in, and label their table of contents. Their desks faced the classroom whiteboard in two long rows, but students often moved these around to fit their needs to see the board and the notes that I wrote on it. Each week we worked on a new Algebra topic, and the week's topic was the focus of the problem solving that I recorded for the purposes of this study. Each topic became one "cycle" which I will discuss in this chapter. Throughout the research that I did in Fall, 2018, I recorded and analyzed students as they worked through four different types of Algebra problems, one in each instruction cycle.

The first week of recording (Cycle 1: Equations), I covered all types of equations with my students. The equations progressed in difficulty throughout the week, but for the recordings, I had students solve basic, two-step equations. The second week of recording (Cycle 2: Inequalities), I covered all types of inequalities. Because these are solved very similarly to equations, I had students solve compound inequalities in the recording to increase the difficulty level. The third week I did not record because we had some catch-up work to do. The fourth week (Cycle 3: Solving for "y"), we had started looking at lines and had to practice how to put the equation of a line (an equation with an x and a y variable used to graph lines) into a form that we could use. The fifth and final week (Cycle 4: Graphing Lines) used what we learned in Cycle 3 to put the equation into the correct form and then graph the line. Data from these four cycles were used to analyze how my students talk about their math process and how I guide them in that process. The notes and my explanations for each topic included particular terms related to solving that type of problem. I hoped that students, in their think alouds, would use these terms.

Table 4.1 identifies the cycles, the various math terms I hoped to hear in the students' recordings, and the students who were recorded in each cycle.

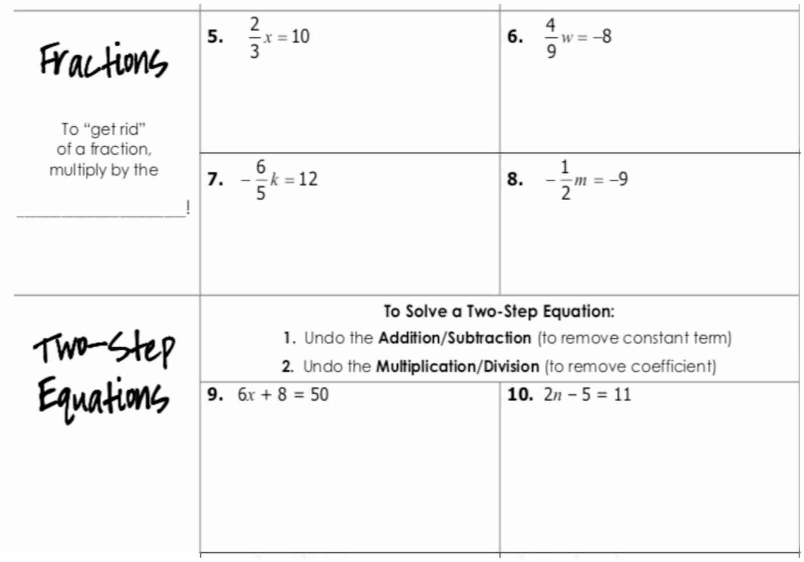
Table 4.1: *Participants and My Discourse Expectations*

Cycle / Topic	**Math terms related to this cycle**	**Student Participants in this Cycle**
Cycle 1: Equations	Opposite operations Cancel Add, subtract, multiply, divide Reciprocal	Nathan AJ Apa Kua Ang
Cycle 2: Inequalities	Greater than (or equal to) Less than (or equal to) Add, subtract, multiply, divide Cancel Switch the inequality sign Shade in the middle Shade on the outside And, or	AJ Apa Kua Ang
Cycle 3: Solving for "y"	Minus/ Subtract the x Can't Combine Like Terms Divide Coefficient Reduce Two negative numbers	Ang Apa-Kua
Cycle 4: Graphing Lines	Minus/ Subtract the x Can't Combine Like Terms Divide Coefficient Reduce Two negative numbers Y-intercept Slope Positive Negative Line	Ang Apa-Kua

Each day that I recorded students talking about their problem solving, I began the class by making sure they were working on their assigned problems. While they were working, I went around the room and, one at a time, gave each student a problem on my iPad to solve. I wanted to record how individual students explained their problem-solving thought processes for my research. When I videotaped students last Spring for my first phase of research, I videotaped the student from the front so that the video could see their paper, too. However, this made it very difficult to see what students were writing or erasing in the video. Their final paper also did not show what they changed throughout the problem solving if they erased anything. In order to see each step as they wrote it and be able to include these written mistakes that were erased, I came up with the idea for the students to write on the iPad and have the iPad record every stroke the students made or erased in a video format. I used an app called "Show Me" that allowed each equation to be on a different page. The app allows me to record when I want to. It records any audio that occurs in the area and the video shows what is being written on the current page. If students were participating in the research, I recorded them, if they were not, I did not record them. So, at the end, I had a think aloud of both audio (student voices) and video (student work) of the students who were participating in the study, and it never showed video or audio of students that were not participating.

Cycle 1: Equations.

During this first week, we worked on solving two-step equations. For the think alouds, I gave each student a two-step equation to solve. They had to first add or subtract to remove the constant term and then multiply or divide to remove the coefficient. Students had previously learned how to solve one-step equations in which only one step (add, subtract, multiply, or divide) was needed. On Monday (9/24), I gave the students their notes templates for one and two step equations (see partial example--Figure 4.1). We used this type of template every time we took notes.



Fractions

To "get rid" of a fraction, multiply by the

________________!

5. $\frac{2}{3}x = 10$

6. $\frac{4}{9}w = -8$

7. $-\frac{6}{5}k = 12$

8. $-\frac{1}{2}m = -9$

Two-Step Equations

To Solve a Two-Step Equation:

1. Undo the **Addition/Subtraction** (to remove constant term)
2. Undo the **Multiplication/Division** (to remove coefficient)

9. $6x + 8 = 50$

10. $2n - 5 = 11$

Figure 4.1: Notes template (Wilson, 2017)

I purchased the set of Algebra I notes, worksheets, and activities from *Teachers Pay Teachers* (teacherspayteachers.com). Sometimes we used exactly what the creator of the notes provided and sometimes I edited them to fit what I want to teach that day. Because of our faster pacing with the block schedule and to move the class a little faster, I sometimes pieced together notes that were intended to be taught on separate days. Even though the templates were not designed with our textbook or with block schedules in mind, I like using them because they help the students stay focused with fill-in-the-blank sections; they let the students gauge how long note-taking will last that day; and it forces me to mediate and use more language when I teach. In this particular example, the students see the sentence, "To 'get rid' of a fraction, multiply by the reciprocal!" Previously, I would have said that sentence out loud, but with these templates, they can go back and remind themselves later. It forced them to write the word "reciprocal." When I ask them what to do in later classes, they have words as well as the process to look at. Before using these templates, I would often have students take notes as I worked example problems for the whole class. Now, I use these templates because they have some fill-in-the-blank explanations, some typed directions, and some examples to solve. I replaced my demonstrations on the board with the note templates because they increase the students' exposure to language about the math processes and act as a mediational tool to help me mediate the students' problem solving.

I helped the students fill these note templates in as best as I could, modeling possible responses on the whiteboard. I wrote what the students had on their paper on our classroom whiteboard, and I alternately demonstrated each step and asked students to provide answers. I began by providing most of the information, and then I slowly started requiring students to chime in with what they thought the next step should be. If the student was wrong, I waited for someone else to call out the right answer, or I helped the original student correct their answer. If I thought students were leaving out important explanations, I had the class explain the process to get that answer. Every time the students took notes, we followed the same procedure. The students were used to calling out steps and explaining their thought processes in this whole-class context.

In our classroom routines, after students took notes and practiced solving problems as a class, they practiced solving the same type of problem on a worksheet (see partial example - Figure 4.2). When students were practicing on the worksheet, they were allowed to work with any student or students in the room. I circulated and worked with students that asked for help. This was often a challenging task for me because it never seemed as if there were enough of me to go around. Students constantly called out my name for help, often frustrating me because I felt pulled in so many directions. Then the students got frustrated when I did not help them "fast enough."

Name: ______________________________ **Unit 1:** Algebra Basics

Date: ______________________ **Bell:** _____ **Homework 10:** One & Two-Step Equations

Directions: Solve each equation. **SHOW ALL STEPS!**		
1. $y - 17 = -37$	**2.** $-12 + k = 18$	**3.** $5m = -20$
4. $\frac{x}{9} = 3$	**5.** $\frac{4}{5}z = 16$	**6.** $3p - 4 = 11$

Figure 4.2: Example worksheet (Wilson, 2017)

On Tuesday (9/25), students got in small groups of three or four students to solve equations together. I wrote an equation on the board and the groups each attempted to solve it. Each group had one marker and a smaller whiteboard. For each equation, the group member with the marker solved the equation while the other group members helped. Once all groups had solved the equation, we solved it together on the bigger, classroom whiteboard and went over any questions. Then the next student in the group took the marker and was in charge of solving the next equation. As the day progressed, I presented more difficult equations. We started with one- and two-step equations, then added in equations involving the distributive property, and equations with variables on both sides of the equation.

On Wednesday (9/26), the students took notes on how to solve more complex equations (equations with no solution or infinite solutions, proportions, and equations with absolute value). We practiced what it would be like to explain every thought one had when solving a problem. Each time students provided a step for our equation, I prompted them to explain further by asking: “Why?” “How do you know you can do that?” “Why do we do this instead of that?” Once I was satisfied with their explanation, we moved on to the next step. After they took notes, I gave them more equations on a new worksheet to practice individually.

On Thursday (9/27), students came in and started working on the equations on the worksheets that had been assigned on Monday and Wednesday. Some were almost finished with both of their worksheets, and some had barely started. The students who were ahead knew that if they finished they should work on their *Aleks* (aleks.com) problems, which provided practice on similar problems. *Aleks* is an online program that allows students to work on problems that their teacher assigns and get immediate feedback if they are correct or not. While completing their work, they were still allowed to ask for help from anyone in the room. They could find a classmate to help them or get help from the classroom aide. It was very uncommon for students to work alone. Every once in a while, I have a student who does not like their classmates enough to work with them, but more often than not, students want to make sure they got the same answers as their classmates or will work with them if they are unsure of the steps. Once they were settled and working, I was able to go to each student have them solve a problem, while narrating it to me. If students were participating in my research, I recorded them to collect the think alouds.

I moved from student to student and asked each one to pause working on their worksheets (usually I helped them finish up their problem first) and solve a new problem for me. Whether they were participating in the study or not, I asked the students to explain to me what they were doing while they were doing it. This week it was my goal to stay silent no matter what—not help the students in any way, so I could see what they were able to do on their own. For the most part, I succeeded in this—only having a few lines of transcription. In a utopian world, my students would have said such things as, "I subtract five from both sides because there is a positive five on the side with the x and in order to move it from that side, I need to take it off both sides." Realistically, I was hoping that the students would say, "I'm subtracting five from both sides to cancel this five." They ended up saying "minus five, minus five." I believe this is because of the way I gave directions. I never asked them to explain their thinking or to explain why they are able to do what they are doing. I basically asked them to narrate what they were writing, and that is exactly what they did. On Friday (9/28), the students took a quiz over solving equations.

For this episode, I had all four students recorded. Ang solved her equation pretty quickly (only 11 lines of transcription – one of which was my comment) and was correct in her answer. I did help her one time, but it was only to help her plug her multiplication fact into the calculator correctly. She did not erase throughout this episode, but she did self-correct once by adding in a negative sign she missed.

In AJ's episode, my only line of transcription was to remind AJ to "tell it what you're doing." I appreciate that AJ was trying to put more reason into his video by using "(be)cause" in his explanation. He did this when he added more information in the simplifying steps than other videos. He explained, "take off eleven cause they're both the same (Table 4.1, line 10)." Even though I did not ask him to give justifications, he naturally explained why he was able to cross off the 11's by using "(be)cause."

Excerpt 4.1: *AJ Cancelling 11's and Explaining by Using 'Cause*

line	Transcription	Notes	Student Work
10	AJ: Take off eleven cause they're both the same	Explaining why the 11's cancel	$2x - \cancel{11} = 5$ $\cancel{+11}\ \ +11$
11	Add these	("these" means 5+11)	

12	Makes sixteen equals x over two	He says x over two even though that is not what he means. He writes from right to left.	$2x - \cancel{11} = 5$ $\cancel{+11}\ \ +11$ $2x = 16$

AJ also had an interesting part of his video (Excerpt 4.2, lines 13 – 20) where he is working on the last step to solve for x. In line 13 he says "make a line, put two" instead of telling what operation he is using (divide). Again, in line 18 he says, "do…add them," even though his operation is still division (I think this is because he is counting by two's up to sixteen, so he is adding two to figure out the division problem). He finally does say divide in line 20, but this is after he has figured out that 16/2 = 8 and written his answer, x = 8. He writes in division symbols by the 16 and the 2 as if he is going to reduce it instead of solve it, then immediately crosses the division signs off. This is interesting because it sounds like AJ does not know what operation to use because he is giving his steps an inaccurate verbal description, but when he gets his answer, he shows that he was using the correct operation.

Excerpt 4.2*: AJ's Discrepancy Between Verbal Description and Written Action*

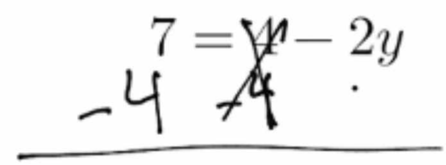
line	Transcription	Notes	Student Work
12	Makes sixteen equals x over two	He says x over two even though that is not what he means. He writes from right to left. His use of "over" is similar to how he used it in the inequalities ensemble.	$2x - \cancel{11} = 5$ $\cancel{+11}\ \ +11$ $\frac{2x}{2} = \frac{16}{2}$
13	Then. Make a line. Put twwooo	I wish he had used the word divide – he neglected to say "divide" later on as well – instead saying add – he does finally say divide in line 20	
14	N: Noob		
15	A: Put two.		
16	Cross these cause they're the same		

17	N: Cross the z's off	He thinks AJ wrote z instead of 2. I am not sure if he is trying to help narrate or if he is just being a goof.	$2x - \cancel{11} = 5$ $\cancel{+11}\ \ +11$ $\frac{\cancel{2}x}{\cancel{2}} = \frac{16}{2}$
18	A: Do…add them (whispers) two four six eight ten twelve fourteen	Wrong operation – he corrects in line 20. I think he means divide, because then he softly counts by two until he gets to 16	$2x - \cancel{11} = 5$ $\cancel{+11}\ \ +11$ $\frac{\cancel{2}x}{\cancel{2}} = \frac{\cancel{16}}{\cancel{2}}$ $x = 8$
19	X equals eight		
20	Divide. By. Nothing	He knows he supposed to be dividing, but he is mixing that up with reducing a fraction. I am not really sure what is going on here.	

Another interesting thing that happened in this video was in line 12. AJ says "sixteen equals x over two" even though the x is next to the two. He is narrating what he writes as he writes it. Because the simplification happens on the right side of the equation in the previous step, he starts by narrating the right side and working his way backwards. So, he says 16 equals x as he writes those characters from right to left, but then he finishes by saying sixteen equals x over two, even though the x is not over two. Here AJ has a verbal "miscue" which was surprising and puzzling, but did not interfere with his problem solving.

In Nathan's Equation video, AJ has 23 lines of transcription while Nathan had 28, which shows that AJ was helping Nathan solve his equation while Nathan posed his questions to AJ. (I was still trying to not give any assistance). It was really interesting to see AJ help Nathan and explain the process using more Yugtun than I could have. This is the only video where the student gets the wrong answer, but I think that is because this is the only video where the correct answer (-3/2) is not an integer. Students tend to think answers are incorrect if they are decimals or fractions and will often do operations differently in order to not get those types of answers. I think that is what happened here because Nathan and AJ were correct up until their final answer. Their work showed they should divide, but they multiplied instead to get their answer (-6).

Excerpt 4.3: *Nathan Mimics AJ's Explanation*

line	Transcription	Notes	Student Work
14	A: [Four-aq] Four-aq (four) and four-aq (four) augarluku (take off)	AJ only tells Nathan to take off the 4's, but Nathan remembers the reasoning AJ used in his video, just minutes ago.	
15	N: Take off four. Cause they're the same		

What is interesting in Nathan's video is how AJ walks him through the problem solving. Since I was trying to avoid providing any help during this week's think alouds, AJ stepped right in when Nathan was struggling and asking questions. AJ leads him through step-by-step, pointing to the screen and answering any question that Nathan asks. Nathan also shows that he is learning from AJ when he gives an explanation that is identical to the explanation AJ provided in his video (Excerpt 4.3). Their teamwork during this video shows the importance of letting students work together even though they might copy each other. There is always the fear that students will take advantage of the setup of the class and just copy their classmates. When students complain that certain students only copy classwork, I remind the class that the purpose of classwork is for them to learn the material before the test. If students choose to copy on the worksheets, they will not be ready for the test and they will fail anyway. I tell students that they should use the worksheets to learn how to do the math so that they will be ready for their test because that is what most important. This teamwork between AJ and Nathan showed that these boys took that message to heart. AJ was able to help Nathan in Yugtun, better than I could in English. Since AJ was able to help Nathan in his first language, and made sure that he understood the process better, so he could be ready to do it on his own. Their collaboration showed the importance of encouraging students to work together on classwork.

Excerpt 4.4: *AJ Helps Nathan in Yugtun to Support Understanding*

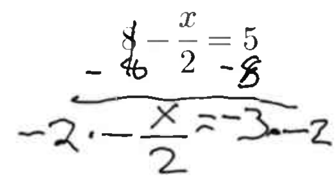
line	Transcription	Notes	Student Work
24	A: Both of *tamarmeng* (Both of) *line-at.* (all the lines.) *Line-ali kitak'* (Make a line now.)		$7 = \cancel{4} - 2y$ $-4 \quad \cancel{-4}$ $3 = -2y$
25	N: *Waten?* (Right here?)		
26	A: Bottom		
27	N: Or *waten*? (Right here?) Long		
28	A: [*aciani* (At the bottom)]		$7 = \cancel{4} - 2y$ n $-4 \quad \cancel{-4}$ $3 = -2y$ $\overline{2} \quad \overline{-2y}$
29	A: two y, I mean two.		
30	A: Negative two y	Even though he says two y a lot, he is trying to tell Nathan not to put the 2y underneath. That only the two is necessary.	
31	N: *Wani?* (Here?)		
32	A: Mhmm		
33	A: Not *tuani* (there)		$7 = \cancel{4} - 2y$ n $-4 \quad \cancel{-4}$ $3 = -2y$ $\overline{-2} \quad \overline{-2}$
34	N: How do you know?		
35	A: *Kiingan* (Only) negative two		
36	N: *Wani?* (Here?)		
37	A: mhmm *tamarmek* (Both of them)		
38	N: *Tua-ll'* (And then) two y?		

In line 24, AJ told Nathan to draw lines underneath the “3” and the “-2y.” Since he did not describe where to draw the lines, Nathan guessed where they should go and gestured to AJ to ask where the lines should go. AJ told him to put the lines at the bottom. AJ then told him to put “2y” which he immediately corrected: “I mean two.” Then in line 30, he further corrected that it should be negative two, but accidentally included the “y” again. Nathan asked in line 31 if he was solving it correctly when he wrote the “2” on the left and the “-2y” on the right underneath the lines. At first, AJ agreed, but then he saw that Nathan wrote the y as well, and told Nathan he was wrong. Nathan erases the “-2y.” When AJ told Nathan “only negative two,” Nathan added a negative to the left side. AJ tells him to do it on both sides and Nathan correctly wrote a “-2” on the right.

Because the two boys were able to work together, they were able to use translanguaging to mediate Nathan’s understanding of the problem. Even though AJ only told Nathan exactly what to do, he was able to help Nathan hear some of the steps in a language that was more familiar to him. It also gave AJ an opportunity to teach someone else, helping his own understanding of how to solve equations.

Apa Kua also solved his equation pretty quickly. He had only 15 lines of transcription, one from another student, and no lines from me. There was one time when he raised his intonation at the end of a sentence as if he were asking a question (see Excerpt 4.5), but I did not say anything and he kept going. There was another student in the background of Apa Kua’s video who watched what Apa Kua did and commented on it (sometimes incorrectly), but that did not seem to confuse or hinder him as he solved the equation correctly and quickly.

Excerpt 4.5: *Apa Kua Asks a Question that Goes Unanswered*

line	Transcription	Notes	Student Work
7	A: Minus two and six over it. Im x over two negative		
8	A: uhhhh. Yeah there's negative right there!	A student in the background was doubting the negative. Apa Kua was defending his answer	
9	A: *Tua-llu* [And then]		
10	A: Times negative two?	Asks a question	
11	A: Times. Negative. Two. Times. Negative two	Keeps going even though I do not answer	

Reflecting on cycle 1.

Throughout this cycle, I noticed many interesting points. AJ explained that he took off the 11s "cause they're both the same." This was interesting because it was the only explanation that went above a simple description. I think I may have modeled why we cross off certain numbers more often than other steps in class, and that was why he felt comfortable giving that explanation. Then, when he told Nathan to take off his 4s (in Yugtun: four-aq and four-aq augarluku), Nathan remembered AJ's explanation and used the same words to describe why he took of the fours ("cause they're the same"). Even though "cause they're the same" is not what I would ideally want students to use as an explanation for this math process, it shows me that AJ saw a pattern when we crossed numbers off, and was confident enough in his pattern noticing to use it as an explanation. His pattern made sense to Nathan, who added it to his explanation as well.

AJ also said some phrases that were surprising such as "x over two" and "add them." I believe this was more of his pattern recognition while he worked to become comfortable with the language. This made me think about instances when students choose surprising language – or words I do not expect them to use.

With Apa-Kua, when I did not answer his question, he was able to continue on without me. When I step back, more of my students' understanding shows. Being able to find a balance between helping too much or too little was something I struggled with throughout this research process and in general in my classroom.

Cycle 2: Inequalities.

During this week, we worked on compound inequalities in which the problem involved more than one inequality. When explaining these to my students, I compare it to a compound sentence in English. In English, a compound sentence is typically two sentences that could stand alone, but they are joined together with a conjunction (and, or, but, so). In math, a compound inequality is two inequalities that could stand alone, but they are joined with "and" or "or." "And" inequalities (Figure 4.3) are typically written as one statement, with two inequality signs

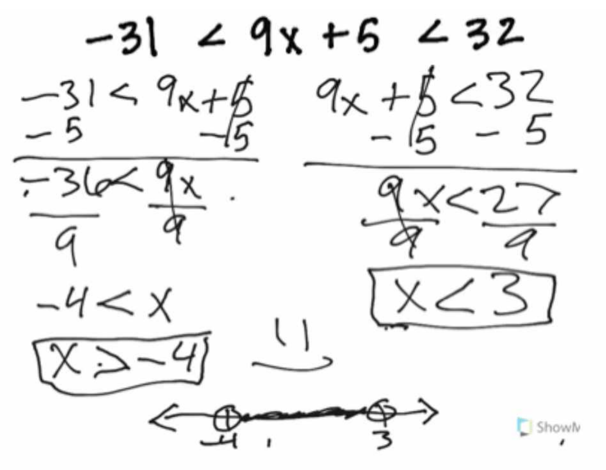


Figure 4.3: "And" inequalities

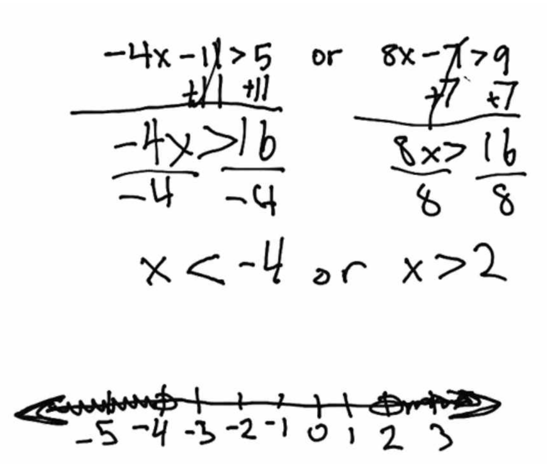


Figure 4.4: "Or" inequalities

that the solver would have to split and solve each side. "Or" inequalities (Figure 4.4) are written as two separate inequalities, joined with an "or," and each side is solved individually. They are typically solved similar to equations, except when multiplying or dividing by a negative. When that happens, the solver has to flip the inequality sign (from a greater than to a less than or vice versa).

On Monday (10/1), we did a discovery activity to learn that rule to flip the inequality sign. To help students discover this rule, I broke the class into small groups and gave each group their own one- or two-step inequality to solve. When they were done solving, I had them pick a number that fit their solution, plug it back into the original problem, and see if it worked. If their answer fit, they put the equation and work on one side of the whiteboard at the front of the classroom. If it did not, they put it on the other side. At this point, they did not know the rule that they had to switch the inequality sign if they divided or multiplied by a negative, so some of the students' attempted solutions worked, and some of them did not work. All of the inequalities where they had to multiply or divide by a negative were on the side of the board that did not work, and all the other inequalities were on the side of the board that did. After we had solved several of both types of problems, I asked the students to look for patterns. They quickly saw that there was a pattern that had to do with negative numbers, but they thought it was about negatives being in the answer. This pattern was not 100% true for all problems, so I asked them to keep going. As they got little pieces of the pattern correct I encouraged them to go further down that path, so we could discover the pattern that multiplying and dividing by a negative made the solution wrong. We then discovered that you could remedy this by switching the inequality sign. I do not know if all students understood why we switch the inequality sign, but they did understand that we should switch the inequality sign.

On Tuesday (10/2), the students broke into three small groups that rotated to a different group every 30 minutes. Depending on what group they were in, the students 1) took notes from me on solving inequalities, 2) practiced on their own, or 3) worked on an online math software. While they were taking notes, I made sure all students knew when to switch the inequality sign, as a review from the rule we had learned the previous day. Each time we got to the part where we might switch the sign, I made sure to ask the students if we should switch it and why or why not. After the 30 minutes was up, they rotated to the next activity or station until all three tasks had been completed.

On Wednesday (10/3), the students solved inequality problems in groups in which I gave students challenge problems and they worked together to solve them. At the end of each problem, one student (chosen randomly) came up to the board and explained to the class how to solve the problem. This was intended as another opportunity for students to practice explaining what they were doing, but typically ended up as me explaining while they solved the problem. This happened because a lot of the students were too shy to explain in front of their peers and refused to talk. When we practiced solving problems this way throughout the semester, students were often unwilling to explain in front of their peers despite being willing to try explaining when it was one on one on the iPad for think alouds. Because they were always willing to explain one-on-one, I did not push speaking in front of the class during cycle two.

On Thursday (10/4), we did the "think aloud" videos just as we had done the previous week. I again asked students to tell me what they were doing while they did it. This time I was not trying to stay silent. I was supposed to offer help only in the form of asking questions, but I was unable to stick to this method. When I offered help, it was typically what I would say naturally instead of following a prescribed rule. On Friday (10/5), we took a quiz.

For this episode, I had three students recorded. During AJ's video, Nathan kept pulling my attention from AJ in order to ask me questions about the problem he was solving on his worksheet. Even though I got very frustrated with Nathan continually pulling my attention from interacting in the think alouds, I kept his comments in the transcript because his comments show a lot about what a typical classroom is like. I do not have a picture for what he was working on, so it is hard to follow his problem. Nathan in particular tends to struggle a lot in math class

Excerpt 4.6: *Nathan's Requests for Help*

Line	Transcription	Notes
65	N: [Do I do this?]	Nathan is trying very hard to get my attention for help. You can hear him in other videos also asking for help.
66	N: What is this called? What is it? Ms. Boyd.	

when he has to work on his own (quizzes and tests), but does very well in class, providing answers during whole class notes, and while doing classwork. However, while he is working on the classwork, he tends to ask for a lot of support like he did in this video. He often will not move on without me if he feels that he is not getting adequate support. His requests for help are interesting because they show a misbalance in my classroom—that either more direct support from me or additional teaching students more agency in the classroom is needed. Even though Nathan was asking for a lot of support, his comments throughout AJ's video were: "I was right!" "Two x?" "Positive or negative?" "Do I put 17 right here?" "So I do this one?" "Not times?" "Do you guys switch it?" "What is this called?" All of these responses show how hard Nathan was trying to solve the problem. He did ask, "Now what?" one time throughout AJ's video, but that was the only time that he asked he to provide the next step. While his questions do not show mastery of the topic, they do show perseverance and enough understanding to narrow down his potential next steps.

Often in class, AJ said that he hates math because it is too hard. Since he took Algebra I, he has told me that he is glad he is not taking a math class because it is too much work. In this video, he only asked for help three times, and when he was not asking, he did not make mistakes.

During class, he would often wait for me or the classroom aide to help him before doing work. In this video, AJ was uncertain at first, but a quick hint from me and Nathan was all he needed to get started.

Excerpt 4.7: *AJ and Nathan's Role Reversal and AJ's Confidence*

line	Transcription	Notes	Student Work
3	A: What do you mean or?		
4	T: Solve this one. Solve this one. Remember like yesterday?	I pointed to the first inequality (-4x-11>5) then the second one (8x-7>9) to show AJ that he should solve them separately.	
5	N: Four! Eleven! … You put eleven.	Nathan helps him – it is good to see Nathan gaining confidence.	$-4x-\cancel{11}>5$ or $8x-7>9$ $+\cancel{11}\quad +11$
6	A: [Okay] [eleven plus]		
7	A: eleven plus. Make a line. Cross off this		
8	N: [how do you know? I was right!]		
9	A: one. Um… add this one. Sixteen. This *augkuciq*. (this kind). X. Four. Negative.	He is working backwards from the 5+11 to get 16. When he says "This *augkuciq,*" he is referring to the greater than sign, then he lists the things that he is bringing down from above from right to left.	$-4x-\cancel{11}>5$ $+\cancel{11}\quad +11$ $-4x>16$

Excerpt 4.8 highlights my observation that Apa-Kua repeatedly uses "okay" as a transition word. I connected it with a story he had told me earlier. Apa-Kua told me a story one day after school about how in the Yupik language, "tua-ll'" means "and then" and functions as a

transition word. He showed me a video of when he was little and told stories he used “tua-ll’” many times in all of his stories. In English, this was as if he told one action of a character, “and then” another action, “and then” another action, “and then” another action until the end of his story as if it was one long run on story. I believe that is what he is doing when he says “okay” in the math problems. He is talking through this math problem as if it was a story from his

Excerpt 4.8: *Apa Kua Uses “Okay” as a Transition Word*

Line	Transcription	Notes	Student Work
8	okay minus five on both sides	I have noticed that he says “okay” a lot. I think he uses it as a transition (see note for longer story).	$-31 < 9x+5$ $-5 \qquad -5$
9	okay make a line, ok- (laughs)		

childhood. But instead of “tua-ll’,” I think his English version is “okay.” I was talking to him after I transcribed this video, and I reminded him of the video he showed me and told him that I thought he uses okay for that same purpose in English. Once I explained a little more with an example he laughed really hard (I assume because the connection made sense to him). To me, his laughter was another layer of accuracy to assure me that his use of okay is very similar to his Yup’ik use of tua-ll’. This suggests to me that narrating math problem solving may, in some ways, feel to the students as if they are telling a story.

In Ang’s video, I noticed that I coded with the phrase, “prompting with a question to self-correct” several times while trying to categorize the type of help I am giving. Sometimes, my prompts to invite students to re-examine their work and to self-correct work out flawlessly. For example, in this example, I repeat back the math fact that the student should be doing, she

Excerpt 4.9: *Ang’s Correction*

Line	Transcription	Notes	Student Work
2	A: Negative one… eighteen right?	She does not say the negative on the 16, but she does write it without prompting	$-2x+1 < -17$ $+1 \qquad +1$ $-2x < -16$
3	T: Uhh… Negative seventeen plus one?		
4	A: [oh] oh it’s sixteen		

realizes her mistake, and immediately corrects it. In later concepts (solving for y and graphing lines), my prompts do not always have the effect I wanted them to. In these more challenging topics, there were times when I prompted the student to self-correct, and they did not understand my prompt or they did not know what to do next. When this happened, I either questioned them several times in different ways, trying to help them figure out what they did wrong, or I immediately provided the correct answer more directly without pushing them to try harder or breaking down the problem more. From this pattern, I noticed that I struggle with knowing how much help to give to my students. I do not want them to rely so much on me to correct everything for them, but I also do not want them to feel lost or helpless.

Reflections on cycle 2.

In this cycle, I started to see students make more mistakes than in cycle 1. This topic was more difficult than last week's, and it showed in their think aloud videos.

Since I was helping more naturally this week, and the topic was not too difficult yet, I noticed that when I prompted students with a question to self-correct, they did pretty well. Even though Nathan repeatedly asked for help during AJ's video, the help I provided to him was more scaffolding than process explanation. For example, he would ask mostly yes/no questions (i.e. "Do I do this?") or questions that I could respond to with one word (i.e. "What's this called?"). He did not ask me to explain the process to him. This makes me hopeful for being able to find a balance in how much help I give my students. I would much rather they focus on asking scaffolding rather than process questions to develop their agency.

AJ showed that, despite his verbal opposition to math, he typically did not need as much scaffolding as his peers. Ang also responded to every prompt I gave her almost immediately. Since this topic was still fairly straightforward for the students, I wish I had asked for more explanations from them. If I had, I think I would have been able to delve into what they understood and how they could articulate that understanding more.

Apa-Kua's narration made me curious about the story-telling aspects of math. He might not have intentionally used "okay" as a transition in his problem, but it sounded like the transitions he used in his Yugtun story telling as a child. Because I wanted to look into how my students talk about their math, this made me wonder if a connection between storytelling and mathematical problem solving would help them in their math classes.

Cycle 3: Solving for "y."

The next week of instruction, we worked on catching up on work and making sure students had a grasp of solving equations and inequalities. Students seemed to be very frustrated during the quiz that we took on October 5th, so we spent the next Monday, Tuesday, and Wednesday (10/8 – 10/10) playing equation games and reviewing before taking a chapter test over equations and inequalities on Thursday (10/11). We did not record anything this week, and I did not monitor how I provided feedback to the students.

During the next week, we worked on taking a linear equation that is written in standard form (Ax +By = C, where A, B, and C are integers) and converting it to slope-intercept form (y = mx + b, where m and b are rational numbers – integers or fractions). In order to do this, students had to take their original equation and solve for "y" or get y by itself using all the rules that apply when solving equations. Typically, this process has what I consider to be four steps:

1. Subtract the x's from both sides
2. Simplify by crossing the x's off and leaving only the y's on the left, and writing the x's with the constant on the right (By = -Ax + C)
3. Dividing everything by the coefficient (number) in front of the y
4. Simplifying again (leaving the coefficient in front of the x as a fraction or a whole number - not a decimal) y = -A/B x + C/B

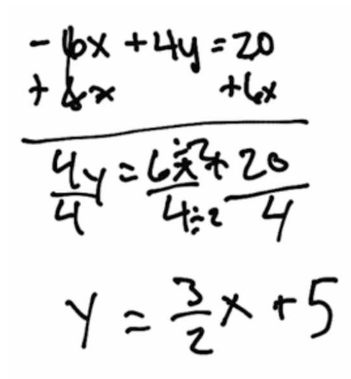


Figure 4.5: Solving for "y"

This week had some kinks to the schedule and we were not able to follow the weekly structure like I had wanted to (standardized tests, impromptu assemblies, students needing more time to just practice concepts, etc.). To prepare students for this topic, we practiced solving literal equations during our equations unit (9/26-10/12). Literal equations are taking an equation with multiple variables and solving for one of them, which is just like converting from standard form to slope-intercept except, converting from standard to slope-intercept is more specific. The problem always starts off as with one type of equation (standard form: Ax + By = C) and is always solved for y (slope-intercept form: y = mx + b). In literal equations, the equation does not have to start off a certain way as long as there is more than one variable. We then took notes on converting from standard form to slope-intercept and had some time to practice it individually (10/15) before working on the think aloud videos (10/16). The students again were working on worksheets while their classmates and the classroom aide helped. I allowed myself to help students naturally even though I was supposed to only answer if they asked me a question.

For this episode, I only had two students recorded. Ang made quite a few errors in her problem, but at first, she was not hesitant in her solving. She was solving without caution—trusting herself to be correct, even when she was not (Excerpt 4.10). There is no question in her

Excerpt 4.10: *Ang's Confidence*

Line	Transcription	Notes	Student Work
7	A: Okay, so one x	Incorrect - Attempting to combine the terms	$\begin{array}{l} -2x - 4y = -12 \\ +2x \qquad\quad +2x \\ \hline 1, \end{array}$

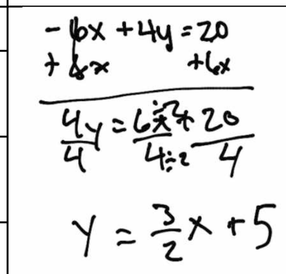
Excerpt 4.11: *Ang's Loss in Confidence*

Line	Transcription	Notes	Pictures
47	T: What's negative twelve divided by negative four?	Prompting with a question	$\begin{array}{l} -2x - 4y = -12 \\ +2x \qquad\quad +2x \\ \hline -4y = 2x - 12 \div \\ \hline -4 \quad -4 \quad -4 \\ y = \frac{1}{-2}x + 3 \end{array}$
48	A: Three?	Guessing answer	
49	T: Mhmm	Confirming	
50	A: What?	Questioning what to put	
51	T: Positive or negative?	Providing two options (she writes plus)	
52	T: Mhmm! Good!	Confirming	

voice. Towards the end of her problem (Excerpt 4.11) she stopped trying on her own—she started waiting for confirmation from me. From that point on, she is doubtful and hesitant and does not write until I have guided her to the correct answer. By this point, I am talking more often than she is, especially in line 51 where I ask her a question and she does not even respond in words, but only writes her response. I am curious about what triggered the seeming change in her confidence. What, in my responses to her, might have triggered this change? Was it something in the problem itself? Or was she frustrated from being corrected so much that she stopped putting herself out there?

Apa Kua, on the other hand, did really well in his video. He did need prompts a couple of times, but anytime I asked him a question, he was pretty good at taking over from there. I always like looking at how I work with this student because he does interesting things and he is pretty good at solving the content on his own.

Excerpt 4.12: *Apa Kua Gets "Help"*

Line	Transcription	Notes	Pictures
26	A: Plus. Twenty?	Attempting to finish the last step	
27	T: What does it say?	Prompting with a question to self correct	
28	A: Twenty divided by four.	Answering the question	
29	T: Mhmm! What's twenty divided by four?	Confirming Prompting with a more specific question	
30	A: [five!]	Answering	

My frustrations from Ang's cycle 3 video is that I do not feel like I was helping her to the best of my ability. When she was solving the first step and could not remember how to move the x's to the other side of the equation, we spent 10 lines, just trying to remember the process of -2x and +2x canceling each other. The questions that I was asking were not helpful for her to figure out how to proceed in solving the problem. Even more frustrating as I reflect on my "help" is that we had lost focus of the goal for the problem. If we are trying to solve for y and the first step is to move the x's over to the right side of the equation, there should be no confusion that this takes them off the left side of the equation. If I had brought her back to the focus of "what are we trying to do" instead of "memorize these steps and try to recreate them," she might have had more success with the problem.

This video in particular made me feel like I am not only encouraging them to use me as a crutch, but training them to use me as a crutch when I do not help them by asking "why" questions or I eventually give up and just give them the answer. I did not like taking the pen away from her either. It took away her power as the student learner and showed her that if she is wrong enough times in a row, I will not take the time to help her get there. As soon as I showed her the correct steps, she immediately recognized it (cycle 3 - line 16), but I still wish that this video had gone differently. When she had to solve for y in the next cycle, she did so flawlessly, so I do not think her struggles crippled her with this type of problem, she only needed more practice with it. To me, the bigger let down is how ineffective my help was.

Reflections on cycle 3.

In cycle 3, I had two very different think aloud videos to reflect on. Ang struggled a lot in her video and really made me question how I give help when my students are struggling and how I should do this differently in the future. Apa Kua helped me realize that it was not all bad. He was able to take my questions and do really well with that feedback. During this cycle, I did not really focus on how my students talk about their math process, but I did focus a lot on how I mediate their problem solving. I was left with a feeling of incompleteness for Ang's problem solving, and a desire to do better for other examples like this.

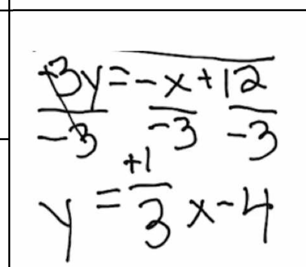
Cycle 4: Graphing lines.

During this week, we took last week's topic one step further and graphed the equations after putting them into slope-intercept form. To graph the equation, once they had gotten the equation in slope-intercept form ($y = mx + b$), Students had to find the y-intercept (the value of b where it existed on the y-axis) and plot that point. They then took the m value and wrote it as a fraction if it was not already. The numerator (top number in a fraction) became their rise (positive for up, negative for down) and the denominator (bottom number in a fraction) became their run (positive for right, negative for left). If their m was negative, they had to decide if the negative went with the numerator or the denominator. If it was positive, but they could not go up anymore, they would be able to have both the rise and the run be negative because if you divide a negative by a negative, it makes a positive.

To prepare for this topic, students did a discovery activity from *Teachers Pay Teachers* to discover what slope-intercept form was and how it could be used to graph an equation. They struggled a little bit with the amount of words and directions on the paper, but since I was circulating and helping out, the students were able to make discoveries and see the patterns that the worksheet intended. This discovery activity was a packet that led them through graphing by creating a table of values then discovering that the slope and y-intercept had been in the original equation all along. It then prompted the students to explain how they could have graphed the line using only the equation (without filling in a table of values). We then took notes and practiced, did the think aloud videos, and took a quiz.

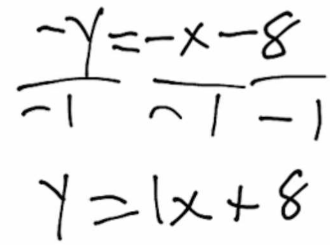
For this episode, I only had two students recorded. Ang, it seemed, made a complete turnaround from her last video because she is still very hesitant to speak in the recording or explain her steps. I did not see a lot of change in this aspect, but she did make fewer mistakes in this cycle than she did in her last cycle. In her third cycle, solving for "y", she struggled to remember the correct process, but in this cycle when she is solving for y again, she does it almost flawlessly. I only had to prompt her once—line 6—and I was able to do it with a question.

Excerpt 4.13: *Ang's Only Prompting*

Line	Transcription	Notes	Pictures
6	T: Even on here you have negative and negative. So, what would your slope be?	Prompting with a question to fix info	
7	T: Positive	(A) Fixing the step (T) Narrating the step	

Apa Kua also did a very good job solving his problem, however, this video made me notice how much of a crutch I was for him during this cycle. Especially here (Excerpt 4.13), he

Excerpt 4.14: *Apa Kua Asks for Confirmation*

Line	Transcription	Notes	Pictures
15	Is it y?		
16	T: Mhmm		
17	A: y. equals negative… one x?		
18	T: Negative divided by negative?	Bringing focus to where the error is	
19	A: Positive? Okay. One?		
20	T: If yo- yeah!	Starting to correct and then cutting myself off (because he is not wrong) - Confirming	
21	A: Plus?		
22	T: Mhmm		
23	A: Eight?		
24	T: Mhmm		

often asked if he was right. Every time I noted that I confirmed for him instead of either asking for him to trust himself or to check it himself. In his last step to solve for y (lines 15 to 24), he asked me before he wrote 2/3 of the characters.

- "Is it y?" (line 15) I confirm (line 16)
- He did not ask about the "="
- "Positive?" (line 19) Even though it ends up being invisible "One?" (line 19) I confirm (line 20)
- He did not ask about the "x"
- "Plus?" (line 21) I confirm (line 22)
- "Eight?" (line 23) I confirm (line 24)

I gave him five directions (confirmations or corrections) for one line of math writing and he asked me six questions.

Reflections on Cycle 4.

Both students were able to solve for “y” and graph the line very well. At this point, my wishes for them were more demanding because I wanted them to keep progressing as they moved on in their math. Ang showed a lot of growth between cycles three and four. I did feel like she could have explained more while she was graphing. Apa Kua also hardly ever need my help even though he asked for it a lot. Even though I noticed myself being over involved before I started this research, I was not able to hold myself back from over-confirming in Apa Kua’s cycle four.

Findings: Teacher Language and Actions

All of the observations that I made throughout the four cycles started to feel very repetitive as I was writing reflections on each cycle. While the students all had different approaches to solving their problems, patterns started to emerge from the interesting parts of the cycles. While I was transcribing and coding each of these different cycles, some of my findings centered on me, and some of them centered on the students. Each finding or pattern is something I noticed for my classroom, but may not be true for all classrooms or even all of the classes that come through my room.

In my first round of research (Spring 2018), I realized that I should have focused more on how I direct students, help them, and talk to them. During my next round of research (Fall 2018), I planned to focus more on how I communicate with my students to mediate their problem solving. While I did not always stick to my original plan of controlling the ways I offer help, I was able to see what happened naturally when I help my students in various ways. From this analysis of my second phase of research, I discovered insights about teacher language and actions that focus on three issues: 1) the way I phrase my prompts or sentence stems; 2) the degree of my involvement in the problem solving; and 3) the role of think alouds as formative feedback. Each one is explained below.

I need to use detailed prompts or sentence stems if I want detailed explanations.

Very often throughout the transcription and analysis process for the think aloud videos, I often found myself wishing that students had explained with more detail or had used more robust math discourse. Instead of narrating what they were doing, I was hoping to hear more explanations for why they were doing the things they were doing. I was happy with the way AJ explained why he was able to cross certain numbers off in Cycle 1, but that was the only instance when I thought that a student was explaining and not just narrating. If students had been able to explain more, I could have asked better questions when I was prompting them to self-correct. If a student was making a mistake, but they were able to verbalize why they thought they were doing the correct thing, I could have asked them more specific questions or prompted them more specifically to find the correct steps.

I do not believe this lack of explanation was entirely on the students. Since I was frustrated with all of the students' descriptions, I knew that they were not at fault. I only asked students to tell me what they were doing while they were doing it. I was asking them for a narration, and they delivered splendidly.

My involvement changes the way students attempt to solve the problem.

How I question students came up a lot, particularly for Ang. When she was able to self-correct right away, I was very proud of our mediation. When she was not able to self-correct right away, I was often frustrated by my responses. At times, I quickly switched over to directly correcting her, and other times we spent a lot of time digging a deeper hole of me asking questions that she did not know how to answer.

When it was done right, I really liked the style of “prompting with a question to self-correct.” I was often assisting the student by drawing their attention to the error, not providing steps or directions for them. Since they were self-correcting, that meant they had at least attempted the problem without me. Yet, I left figuring out the question and realizing what to do next up to them. I was able to mediate for them by breaking down the steps to help them find the correct process, and I liked having that role. What I did not like was my response when the student did not understand my question or was not sure how to fix their error. As soon as it seemed like they were not able to fix it right away, I jumped in and provided more information or more basic questions instead of asking deeper questions, providing more think time, or allowing them to discover their mistake on their own.

For Ang, I wish I did not feel the need to step in as much. If I wanted to help her develop her sense of agency in math, I wish I had been able to prompt that more. When I prompted her visually (like when I took the pen from her and crossed off the “-2x” and the “2x”), she seemed to respond better to the visual hints instead of the written hints. I do not like how this problem played out, but, I am not sure how I would have preferred it to go. When Ang was solving for y in cycle three, it was hard for me to keep asking questions and prompting her. It was so easy to show her that one visual que, but taking the pencil away from her felt like taking away her power as a problem solver and a learner. I think sometimes we get caught up in helping students "the easy way" and that is what hurts our students' agency and self-confidence.

It is so easy to answer students when they are asking if they are right. I have noticed in other instances that I will also repeat “mhmm!” for every step a student writes without the student asking for help. I think it has gotten to the point that if I do not confirm students, they think that means they are wrong. A big part of my students' agency that I wanted to develop was their ability to attempt to solve a problem without my help. In the think aloud videos, my students seemed to be pretty good at attempting problems. More frequently, I saw that they could get started and work through a problem as long as I was sitting next to them, able to confirm every step they wrote down. This brings up a new goal of agency: For students to be confident when they solve a problem or be able to re-work a problem to find their mistake if they think they are wrong.

One-on-one think alouds with students provide important formative feedback.

Sometimes students are a bit over-dramatic with how much they are struggling. I had been grouping AJ in a lower group because he was always telling me in class that he did not understand and he would always ask for help. But in the videos, when he was doing the think alouds, he was solving problems just fine, even when I was not helping. Sometimes, he would even do better than other students (cycle 1, line 10) so I moved him into a more advanced group, and he did just fine. He did not fall behind after I moved him.

I decided to keep Nathan's confusing comments in AJ's cycle two video because it shows that teaching is not nice and neat. Teachers are frequently juggling much more than just two kids asking for help at a time and often have multiple things running through our heads at any given moment. It is hard to tell if he does not know the material as well as I think he does or if he just lacks the confidence to do it on his own. He asks good questions while working and wants to use the appropriate vocabulary, but he shuts down easily if he feels he is stuck. I did not record him again after this, but I have seen him shut down easily in class unless someone is right there making sure he is doing it correctly. When teachers are focused on more than one student at a time, especially if one of the students is pulling their attention more, it is hard to focus on each student's abilities and what they are capable of.

Because of how well AJ was doing in these videos, it seemed as if he really did know what he was doing and that his timid nature in class was just a reaction to low self-confidence or wanting to be sure he is right. In the first cycle, solving equations, AJ not only did his own equation on his own, but he was confident enough to help Nathan solve his equation. In the solving compound inequalities video, AJ solved his problem with my attention divided from him. His verbal reactions to math have made me put him in lower groups before, but after looking at these videos, I started looking for other causes of his protest against math and his ability to solve the problems. He does struggle with some of the vocabulary, but I think the processes are easier for him than the vocabulary. While spending time with the students taught me a lot about their capabilities, spending time analyzing their think alouds taught me even more.

Findings: Student Language and Problem Solving

While I was analyzing the think aloud videos, students often surprised me with the things they said, the connections they made, or the importance of the help they were able to give each other. Without analyzing and spending a fair amount of time with the data, I do not think I would have noticed many of the intricacies of what was happening in my classroom. In the past, I had been quick to write these students off as challenging and sometimes lazy, but looking in depth at what they were able to do when problem-solving helped me see more agency than I had originally thought possible.

Students' language sometimes did not match their problem-solving steps.

In Two-Step Equations, when AJ says "over" (x over 2), but in reality, the x is next to the two. In Compound Inequalities, he says "over" instead of "is greater than." So, that use of the word "over" specifically seemed strange to me because he used it both times when he either did not know what word to use instead or he was just using a filler word. It is interesting because he used "over" in two different scenarios to describe two different types of things that are happening. It obviously does not hinder his ability to solve the problem, because both times he solves the problem really well, but would it help students to have a filler word? Because not knowing those words did not stop him from being able to do the math. Would a filler word help or would that make things more confusing?

AJ also uses "add" when he is dividing (cycle 1 – line 18). His use of "add" even though he was supposed to divide is interesting because he did end up dividing. The math fact he was trying to do was 16/2 to get the final answer. One possible explanation for this instance is he said to add "them" because he is explaining that he is going to skip count by two until he gets to 16. Another possible explanation for his verbalization is that he knew he was performing an operation and just mislabeled it. He performed the correct operation to get the correct answer, but the verbalization is not clear or descriptive to the listener.

Again, these miscues do not affect his ability to solve the problem because he does divide when he is supposed to and he know how to shade the inequality graphs without verbalizing "is greater than." In my analysis of this "miscue", it seems like AJ is using a placeholder word. I am hesitant to call his use of "over" and "add" errors, because there are several reasons why he may be using these words. His use of "over" is not "correct" in either instance (during cycle one or cycle two). He uses it to replace "next to" in cycle one and "is greater than" in cycle two. His use of “add” could be a placeholder for “divide,” but since he does not explain himself, this is just speculation. It makes me think it might be helpful for students to have a filler word for when they do not know the correct discourse to use. On the other hand, I want them to be confident in the math discourse they use and having “filler” words available to them might just give them a reason to not pay attention to the correct vocabulary.

Transition words may have a role in math language, too.

When Apa Kua uses “okay” and other filler or transition words, I think one possible explanation is that Apa Kua has recognized that Algebra has steps just like a story has narrative structure. It is as if he is narrating, “Apa-Kua writes down the problem and then Apa-Kua subtracts five from both sides and then Apa-Kua draws a line and then Apa-Kua figures out that negative 31 minus five equals negative 36.” Another possible explanation is that he is using “okay” as a way to have verbal think time. Maybe that is what he used “tua’ll” for as a child, but it is interesting that something he did while telling a story also appears when solving a math problem. Maybe Apa Kua is not using tua-ll’ as a connection from his childhood in telling stories, but either way, I wonder if there is a way to connect math processes to the cultural art of Yupik storytelling. Because storytelling is so ingrained into the Yupik culture, it could be interesting to try and teach math as a story and see if students responded to it better.

Translanguaging may play a supportive role.

Nathan was very uncomfortable with solving his problem on his own, so AJ stepped in and helped him and they were able to use Yugtun and English simultaneously to help Nathan solve his problem. They are able to switch back and forth between the languages as needed. Even though AJ is telling Nathan what to write, he is using Nathan’s first language and answering every question Nathan asks him. This gave Nathan another chance to hear the steps in a language he is much more familiar with. AJ is not pressing Nathan to solve the problem on his own, but he is reiterating the steps for Nathan and helping Nathan have more exposure to this type of math problem so he can become more independent as he goes.

Conclusion

From my findings throughout the four cycles, I learned a lot about myself as a mediator and my students as problem solvers. Mostly, I was left with a desire to fix my actions and with pleasant surprise with my students' ability to persevere. I decided that I want to be as detailed as possible with prompts or sentence stems if I want my students to communicate in detailed and specific ways. I want to have more phrases ready to use with my students to help them break down the problem and prompt them to self-correct better. Mostly, I noticed how helpful think aloud videos are when trying to understand students' thoughts and misconceptions. For my students, I noticed that they do not always use the correct words when explaining their thoughts, and this does not seem to hinder their problem-solving skills. However, with the increased focus on language in problem-solving for the common core standards, I should continue to help my students develop their mastery of the language of math. I could use math as a story-telling procedure to help my students gain interest in it, and I should let them mediate for one another. Especially with bilingual students, the more I let them explain math processes to each other, the more prepared they will be to understand it.

Chapter 5: Implications

This teacher action research project has changed the way I interact with my students as they solve math problems. It has especially changed the way I listen to what they are saying about their problem-solving process. After more than a year of preliminary research and reflection, I finally focused on these research questions to guide my inquiry:

- How do my students talk about their math process?
- How do I mediate their problem solving?

In this chapter, I explore these findings in more detail, including the implications for both research and for my teaching.

Throughout the research process, my findings brought up many questions for my future practice and other future research. Some of those questions are detailed in Table 5.1 below:

Table 5.1: *Findings, Questions, and Implications for Future Research and Practice*

Findings from the Analysis	**Implications for Teaching Decisions**	**Questions to Guide Future Research**
Students' explanations were brief and often incomplete.	Use detailed prompts or sentence stems to demonstrate to students how the language is used.	What other ways can I coax more detailed explanations from my students?
My questions can provide feedback to help students decide what to do next.	Prompt with a question to suggest the next step.	What language can I use to help my students break down their problems?
My questions or prompts can confuse students or do too much of the work for them.	Try to help only when students cannot continue on their own. Make questions direct and straightforward.	What types of questions are best to prompt students when they get stuck or are unsure?

Working one-on-one with students while they think aloud helps teachers learn about their capabilities and what help they need.	Work one-on-one with students and pay close attention to what they are saying about their process.	What other resources or tools help teachers learn about their students' capabilities?
Math can be compared to other aspects of my students' lives, such as Yup'ik storytelling.	See if students connect to a storytelling aspect of mathematical problem solving.	Should I use this aspect of the Yup'ik culture in math? Would it help my students or create unnecessary confusion?
Students' explanations sometimes did not match their actions.	Encourage students to listen to themselves and think about their language choices. Keep working toward a mastery of math discourse.	How can content teachers teach their students to emphasize the language component and prioritize learning and using the vocabulary?
Students are able to help each other learn by combining their first language with English as they communicate with each other.	Translanguaging is very important for bilingual students and should be used as authentically as possible as often as possible.	How can teachers integrate translanguaging in the most authentic ways?

These implications and questions suggested six over-arching questions, which I address below.

What Should I Say to Get the Responses I Want?

When I asked students to tell me what they were doing in their math problem, they narrated the process instead of explaining. When I was setting up my research, I was hoping they would explain it more, but because I said to them "tell me what you're doing" they did exactly that and narrated their writing. So, this asks the question, what *should* I have prompted them with to get a more desired response?

I think if I had offered more detailed prompts or example sentence stems for the type of explanation I was looking for, the students would have delivered on that as well. I wish I had asked the students, “Explain to me why you are doing what you do,” or if I gave them sentence stems for their explanations, I would have gotten better results. I wish I had structured my help more so that I could have had better results. I think that, as teachers, we need to remember that we need supports too. The questions that Johnston (2003) lists to develop agency (Table 2.1) could have been posted on my wall to help me stay focused. Additionally, I could have looked into other resources to help me mediate with my students. While my note templates (Figure 4.1) and think aloud videos helped me set up my research, I could have looked into more avenues for more focused mediation. Patterson, Wickstrom, Roberts, Araujo, and Hoki (2010) list many instructional tools that can be used for mediation with students (pp. 12-13). While not all practices easily translate to math, I know there are resources they list that I could have used to further mediate for my students (i.e. student choice, informal conversations, assignments that use funds of knowledge, word walls, and multiple modes to mediate knowledge). Being able to mediate more intentionally for my students would help me figure out when to provide more help and when to back off.

How Much Support Should I Give When Students are Struggling?

At times, I would negotiate with my students to help them self-correct and other times I saw the students struggling and gave more direction. It was hard for me to find the balance between giving them too much support (which was the problem I saw that sparked my research) and not giving them enough support (which could increase student frustration). So, I wanted to look into how much support should I give as I try to find that balance and develop my students' agency.

Because Ang struggled so much when solving for y in cycle three, I wonder if a better way to help her would be to back up and have her work a simpler problem and then progress, but that is not always possible in a timed math class. This makes me wonder, how should we assist students when it seems like they are completely lost? Are there better ways to scaffold for them without doing the problem for them? In a future TAR, I might generate a set list of questions or sentence stems to see if certain questions could help students progress.

In Morrone's 2004 study, she found that some students "may need constant affirmation that they are making progress toward achieving the 'right answers.' The instructors' unwillingness to provide these answers may have contributed to an increased sense of frustration for these students" (p.28). The students were frustrated with the uncertainty of the social math they were asked to do because they wanted there to be one right answer – not a myriad of possible paths and solutions to their application problems. This is a common attitude towards math, whether working on word problems or procedural problems. Students want to know if they are doing it right because they do not want to start over. Some students will refuse to try a problem is they are not completely sure of the "correct" process to solve it.

Several people who get anxious in math, find it comforting to have someone confirming them every step of the way. I think this is why I started confirming them so much. I do not want to create more math anxiety for my students, but I also wish there was more value in trying, getting something wrong, and going on a discovery quest to figure out why it was wrong without the anxiety. It is hard to find the balance between letting the students rely on me too much and nurturing them enough so that they can trust themselves or think of themselves as "math people."

I think this pattern of finding the balance between too much spoon-feeding and throwing kids in the deep end is a big one that does not have a right answer. Morrone (2004) found that the instructor in her study pressed the students first to challenge them. Only when the students were unable to proceed or their answer was incomplete, the teacher would step in with more scaffolding and developmental questions. After scaffolding, the teacher would always increase the complexity of their next question (p. 31). This allowed the teacher to support where needed, and keep the rigor and expectations of the students high. This balance is very difficult to achieve, but it forces teachers to ask themselves: How much is too much help? Does it change from student to student? How do you allow students to be independent without making them want to give up?

What am I Learning About my Students as they Verbalize their Thoughts?

When I watched and listened to each student solve a problem individually, I was able to discover a lot about them. Just through the process of these think alouds, I was able to understand my students learning and what they were capable of a lot better. I thought it was really important to look at what I was able to learn just by sitting down with each of the students in my classroom and devote my attention to them while they verbalize their thinking.

The process of watching students solve a problem while I focused on them was nice and helped me to be able to work one-on-one with students to see how they solved problems. Otherwise, I would rely on what students told me about their mathematics abilities or how often they ask for help. It helped me to be able to push AJ more and see that he could handle it. AJ showed me that I cannot take a student's word for their amount of struggle. I already did not pay much mind to their test scores, but I typically would listen if they told me they did not understand or needed help. I have started to require students to try more often, rather than just jumping in and helping. This has helped me demand that the students try before I jump in and help them correct their work. The use of TAR and think alouds in my classroom was a huge eye-opener for me because it made me challenge my thoughts about myself as a teacher and my assumptions about my class. Analyzing my students' work using these two methods of noticing what happens in my classroom, allowed me to systematically look at what I could do to create positive change.

Additionally, the think aloud strategy helped students become more comfortable with their math problem solving process. Having someone to work through the problem with them, helped my students persevere. They never said, “I don’t know what to do” in the think aloud process, but when working with pencil and paper my students often start off with, “I don’t know how to do it.” The additional layer of students being able to show their knowledge verbally helped some students who are not as comfortable with math or mistakes. Both the writing on the iPad and the words that they said out loud did not feel as permanent to them as what they would write with pencil and paper. By helping students become comfortable with mistakes, they are more likely to attempt the problems and increase their agency.

How is Problem Solving Similar to a Story?

My students use words and phrases like "okay" "let's do this" and "alright" as almost a transition in their problem solving like a story. In Yugtun there is a word "tua'll" which translates to "and then" and serves as a transition word. Apa Kua showed me a video of him as a younger boy telling a story in Yugtun in which he would say a sentence "tua'll, and then" tell another sentence, "tua'll, and then" over and over until his story was over. I saw similarities to how he solved his math problem when he would do a step, and then say okay. Do another step, "okay" and so on. It seemed like he was telling his math problem like a story. This made me wonder whether students see math problem solving as a story, with specific plots or steps and if not, would it help them if I tried to guide them to that connection?

Storytelling could help my students turn their narrations into explanations. In stories, the author does not simply state steps, but they explain why the steps are happening. This connection could help my students explain their math steps and see that the explanations are just as important. While I'm still not sure how math could be taught with storytelling, the concept of bringing my students' culture and experiences into the classroom has merit no matter what aspect is being brought in. For my indigenous students, language, culture, and experiences can all assist in their learning experience.

What is the Importance of Student "Miscues"?

Sometimes my students said words that did not really fit what they were writing, but it did not seem to hinder their ability to solve the problem. So sometimes they used a word that did not seem to fit what they were writing, but they still solved the problem correctly. At these times, I was surprised by their word choices and unsure if I should correct them.

So, this raises the question to me, does it really matter if what he says is correct if he is able to solve the problem correctly? This is balanced by, since the common core has an increased emphasis on language and reasoning, should we make sure that our students have a proficient command of the content discourse? Recognizing the importance of discourse, if there are simple procedural miscues in what students are saying that do not hinder their ability to solve the problem, I think those errors might not significant, but it is something that I need to explore further.

When AJ used surprising language, I wonder if he is even listening to himself or self-monitoring? It seems to suggest the need for further research in which the students would solve the problem, and then to record their explanation/justification of the problem solving. Once the problem was solved, they could focus on the language that describes their process. On the other hand, maybe we should expect these "mistakes" during the initial problem-solving think aloud as it is natural to say words that do not always match our actions. This makes me think it would be better if students had a process to go through in order to figure out the correct discourse if that is important for their problem-solving process, and what would that process look like?

How can Peers Help When Given an Opportunity for Translanguaging?

Students are able to help each other learn by combining their first language with English to communicate with each other. Since the majority of my students do have Yugtun as their first language, it was really interesting to be able to see them help each other in their first language. When students are given the opportunity to do this in the classroom, it is called Translanguaging. Garcia, Johnson, and Seltzer (2017) say, “If we limit students to the use of only part of their language repertoires—especially the part that is considered their weaker language—we also limit their ability to learn” (p. 105). By allowing students to communicate in whatever way is effective for them and teaching them to access the full amount of their knowledge, we are setting them up for success, even if it is less traditional. As I explore this, I wonder, what other ways can students help each other or how can I direct this help to most benefit the students?

When peers are supporting each other, I see that being able to mediate for my students and prompt them in challenging ways is very important, but it is also important to give them opportunities to mediate for each other. When students are going through the design cycle (Cope & Kalantzis, 2009), after they have learned the new information and have processed it, they are ready to redesign their knowledge. The students remake themselves as they develop their understanding and their skills. Once AJ learned the material, he had redesigned himself as someone who could teach this concept to another person. AJ was mediating with Nathan instead of me, which is a very important part of developing agency for the students. Allowing him to use his newfound knowledge in a new role, helped him continue learning and adapting as he helped Nathan.

Conclusion

Throughout the teacher action research that I conducted in my classroom, I learned a lot, I was left with more questions, and I had a desire to try new things in my classroom. As I try to answer these questions and implications, I have to focus on my students and try to give my students a level playing field. If my students connect to the storytelling aspect of their culture, I should help them use math to tell a story.

Throughout my research, the following statements were my biggest takeaways and how I want to continue my research. When my students give me brief or incomplete answers, I should prompt them more specifically and look into better ways to prompt my students. I should always be mindful of my feedback to help my students break down the problems, but I also need to watch out for providing too much feedback or confusing feedback. In a world of increasing class sizes, being able to work one-on-one with students is even more crucial. I should strive to find time to meet with my students individually. In Bush Alaska, I have many cultural resources available to me that I should try to incorporate. I also have a great resource in my students. They relate to each other better, and they understand each other better than I can. Authentic translanguaging has a crucial role with every bilingual student. Content teachers need to emphasize to their students their content language. Regardless of the subject, every teacher needs to be teaching their subjects' language to their students. While I know I will never be done exploring and researching, this research was very effective in opening my eyes to my students' needs and encouraging me to continue learning and growing.

References

Aleks. Accessed March 2018. https://www.aleks.com.

Bernadowski, C. (2016). “I can’t evn get why she would make me rite in her class:” Using think-alouds in middle school math for “at-risk” students. *Middle School Journal*, *47*(4), 3-14.

Casher, C. L., & Stotler, D. L. (2015). The four resources model: Critical literacy in the time of the common core. *Michigan Reading Journal, 48*(3), 19-27.

Charmaz, K. (2014). *Constructing grounded theory* (2nd ed.). Thousand Oaks, CA: SAGE.

Cope, B., & Kalantzis, M. (2009). “Multiliteracies”: New literacies, new learning. *Pedagogies: An International Journal, 4*(3), 164-195.

Duyke, K. V., & Matusov, E. (2016). Flowery math: A case for heterodiscoursia in mathematics problems solving in recognition of students’ authorial agency. *Pedagogies: An International Journal, 11*(1), 1-21.

Freebody, P., & Luke, A. (1990). ‘Literacies’ Programs: Debates and demands in cultural context. *Prospect 5*(3), 85- 94.

Freire, P. (2005). *Teachers as cultural workers: Letters to those who dare teach.* Boulder, CO: Westview Press.

García, O., Ibarra Johnson, S., & Seltzer, K. (2017). Translanguaging pedagogy in action. In *The translanguaging classroom: Leveraging student bilingualism for learning* (pp. 99-117). Philadelphia, PA: Caslon, Inc.

Jewitt, C. (2009). An introduction to multimodality. In *The Routledge handbook of multimodal analysis.* (pp. 14-27). New York, NY: Routledge.

Johnston, P.H. (2003). *Choice words: How our language affects childrens’ learning*. Portland, ME: Stenhouse.

Kalantzis, M., & Cope, B. (2008). Language education and multiliteracies. In S. May & N. H. Hornberger (Eds.), *Encyclopedia of language and education (Vol. 1: Language policy and political issues in Education)* (2nd ed.) (pp. 195-211). New York, NY: Springer.

Khan Academy. Accessed March 2018. https://www.khanacademy.org.

Kucan, L., & Beck, I. L. (1997). Thinking aloud and reading comprehension research: Inquiry, instruction, and social interaction. *Review of Educational Research, 67*(3), 271-299.

Lantolf, J. P., Thorne, S. L., & Poehner, M. E. (2015). Sociocultural theory and second language development. In B. VanPatten & J. Williams (Eds.), *Theories in Second Language Acquisition* (pp. 207-226). New York, NY: Routledge.

Mckeown, R. G., & Gentilucci, J. L. (2007). Think-aloud strategy: Metacognitive development and monitoring comprehension in the middle school second-language classroom. *Journal of Adolescent & Adult Literacy*, *51*(2), 136-147.

Mills, G. (2018). *Action research: A guide for the teacher educator* (6th ed.). Upper Saddle River, NJ: Pearson.

Morrone, A. S., Harkness, S. S., D'Ambrosio, B., & Caulfield, R. (2004). Patterns of instructional discourse that promote the perception of mastery goals in a social constructivist mathematics course. *Educational Studies in Mathematics*, *56*(1), 19-38.

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). Common Core State Standards for mathematics. Retrieved from http://www.corestandards.org/Math/\

Patterson, L., Wickstrom, C., Roberts, J., Araujo, J., & Hoki, C. (2010). Deciding when to step in and when to back off: Culturally mediated writing instruction for adolescent English learners. *The Tapestry Journal*, *2*(1), 1-18.

Rainey, E., & Moje, E. (2012). Building insider knowledge: Teaching students to read, write, and think within ELA and across the disciplines. *English Education, 45*(1), 71-90. Retrieved from http://www.jstor.org/stable/23365001

Secolsky, C., Judd, T., Magaram, E., Levy, S., Kossar, B., & Reese, G. (2016). Using think-aloud protocols to uncover misconceptions and improve developmental math instruction: An exploratory study. *Numeracy*, *9*(1), 1-16.

Serafini, F. (2012). Expanding the four resources model: Reading visual and multi-modal texts. *Pedagogies: An International Journal*, *7*(2), 150-164.

Shanahan, T., & Shanahan, C. (2012). What is disciplinary literacy and why does it matter? *Topics in Language Disorders, 32*(1), 7-18.

Thompson, P. W. (1991). To experience is to conceptualize: A discussion of the epistemology and mathematical experience. In L. P. Steffe (Ed.), *Epistemological Foundations of Mathematical Experience* (pp. 260-281). New York, NY: Springer-Verlag.

Wells, G. (2007). Semiotic mediation, dialogue and the construction of knowledge. *Human Development*, *50*(5), 244-274.

Wilson, G. (2017). *Algebra 1 curriculum.* Retrieved from https://www.teacherspayteachers.com/Product/Algebra-1-Curriculum-with-Activities-749721

Appendices

Appendix A: Project Approval Letter

UNIVERSITY OF ALASKA FAIRBANKS

Institutional Review Board

909 N Koyukuk Dr. Suite 212, P.O. Box 757270, Fairbanks, Alaska 99775-7270

(907) 474-7800
(907) 474-5444 fax
uaf-irb@alaska.edu
www.uaf.edu/irb

November 21, 2017

To:	Leslie Patterson, Dr Principal Investigator
From:	University of Alaska Fairbanks IRB
Re:	[1155707-1] How math language affects problem solving skills

Thank you for submitting the New Project referenced below. The submission was handled by Expedited Review under the requirements of 45 CFR 46.110, which identifies the categories of research eligible for expedited review.

Title:	How math language affects problem solving skills
Received:	November 14, 2017
Expedited Category:	*6 and 7*
Action:	APPROVED
Effective Date:	November 20, 2017
Expiration Date:	November 20, 2018

This action is included on the December 6, 2017 IRB Agenda.

No changes may be made to this project without the prior review and approval of the IRB. This includes, but is not limited to, changes in research scope, research tools, consent documents, personnel, or record storage location.

America's Arctic University

Appendix B: Amendment/Modification Letter

UNIVERSITY OF ALASKA FAIRBANKS

(907) 474-7800
(907) 474-5444 fax
uaf-irb@alaska.edu
www.uaf.edu/irb

Institutional Review Board

909 N Koyukuk Dr. Suite 212, P.O. Box 757270, Fairbanks, Alaska 99775-7270

July 27, 2018

To:	Leslie Patterson, Dr Principal Investigator
From:	University of Alaska Fairbanks IRB
Re:	[1155707-2] How math language affects problem solving skills

Thank you for submitting the Amendment/Modification referenced below. The submission was handled by Expedited Review under the requirements of 45 CFR 46.110, which identifies the categories of research eligible for expedited review.

Title:	How math language affects problem solving skills
Received:	July 25, 2018
Expedited Category:	*6 and 7*
Action:	APPROVED
Effective Date:	July 27, 2018
Expiration Date:	November 20, 2018

This action is included on the August 1, 2018 IRB Agenda.

No changes may be made to this project without the prior review and approval of the IRB. This includes, but is not limited to, changes in research scope, research tools, consent documents, personnel, or record storage location.

Appendix C: Continuing Review/Progress Report Letter

UNIVERSITY OF ALASKA FAIRBANKS

(907) 474-7800
(907) 474-5444 fax
uaf-irb@alaska.edu
www.uaf.edu/irb

Institutional Review Board

909 N Koyukuk Dr. Suite 212, P.O. Box 757270, Fairbanks, Alaska 99775-7270

November 1, 2018

To:	Leslie Patterson, Dr Principal Investigator
From:	University of Alaska Fairbanks IRB
Re:	[1155707-3] How math language affects problem solving skills

Thank you for submitting the Continuing Review/Progress Report referenced below. The submission was handled by Expedited Review under the requirements of 45 CFR 46.110, which identifies the categories of research eligible for expedited review.

Title:	How math language affects problem solving skills
Received:	October 27, 2018
Expedited Category:	6 and 7
Action:	APPROVED
Effective Date:	November 1, 2018
Expiration Date:	November 20, 2019

This action is included on the November 7, 2018 IRB Agenda.

No changes may be made to this project without the prior review and approval of the IRB. This includes, but is not limited to, changes in research scope, research tools, consent documents, personnel, or record storage location.

Appendix D: Cycle 1 - Ang Solves a Two-Step Equation

$$\cancel{-5} + \frac{x}{-15} = -6$$

$$\cancel{+5} \qquad\qquad +5$$

$$\cancel{-15} \cdot \frac{x}{\cancel{-15}} = -1 \cdot -15$$

$$x = 15$$

Appendix E: Cycle 1 - AJ Solves a Two-Step Equation

$$2x - 1\cancel{1} = 5$$

$$\cancel{+11} \quad +11$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{16}{2}$$

$$x = 8$$

Appendix F: Cycle 1 - Nathan Solves a Two-Step Equation

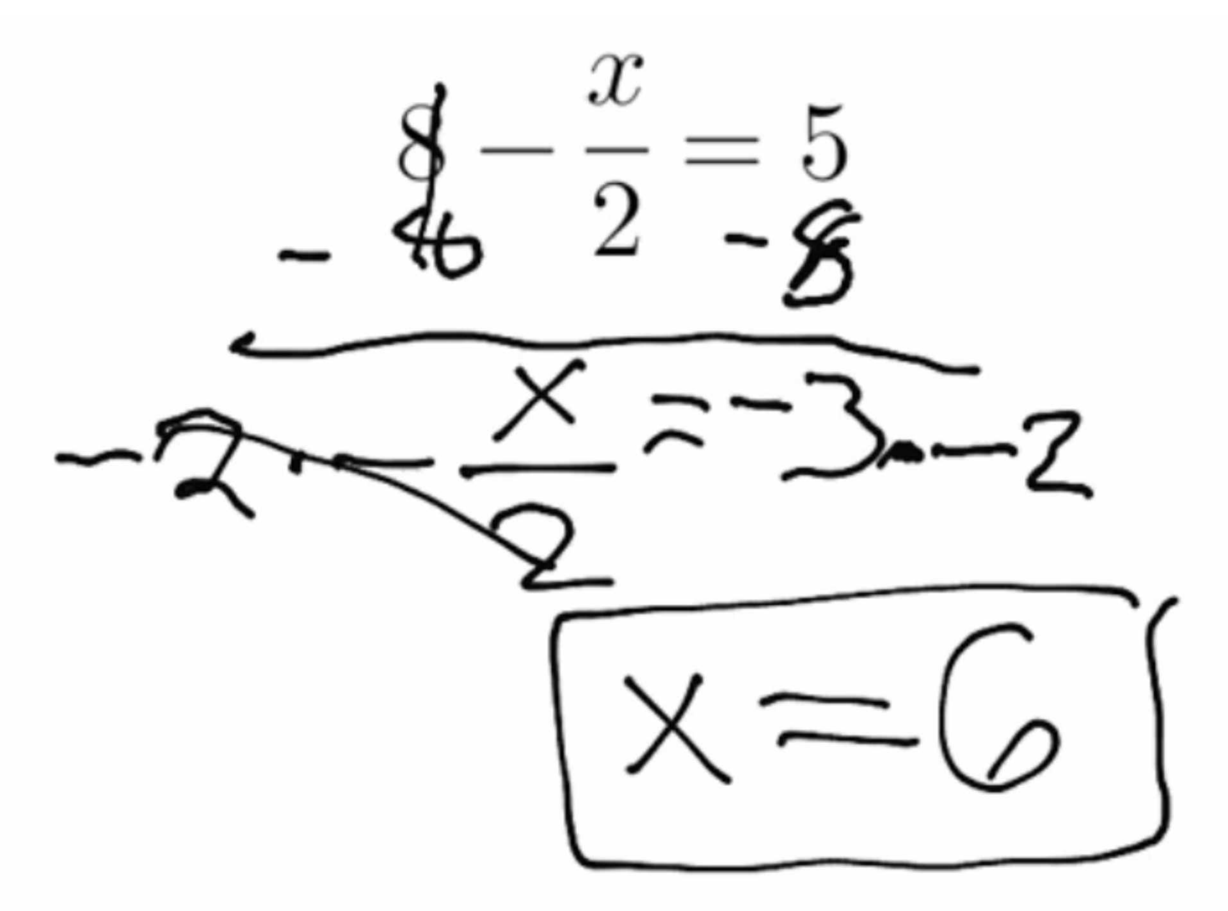
$$7 = \cancel{4} - 2y$$ nube

$$-4 \quad \cancel{-4}$$

$$\frac{3}{-2} = \frac{\cancel{-2y}}{\cancel{-2}}$$

$$\boxed{y = -6}$$

Appendix G: Cycle 1 - Apa Kua Solves a Two-Step Equation



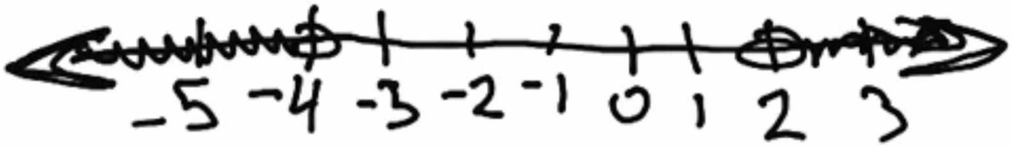
Appendix H: Cycle 2 – AJ Solves a Compound Inequality

$$-4x - \cancel{11} > 5 \quad \text{or} \quad 8x - \cancel{7} > 9$$

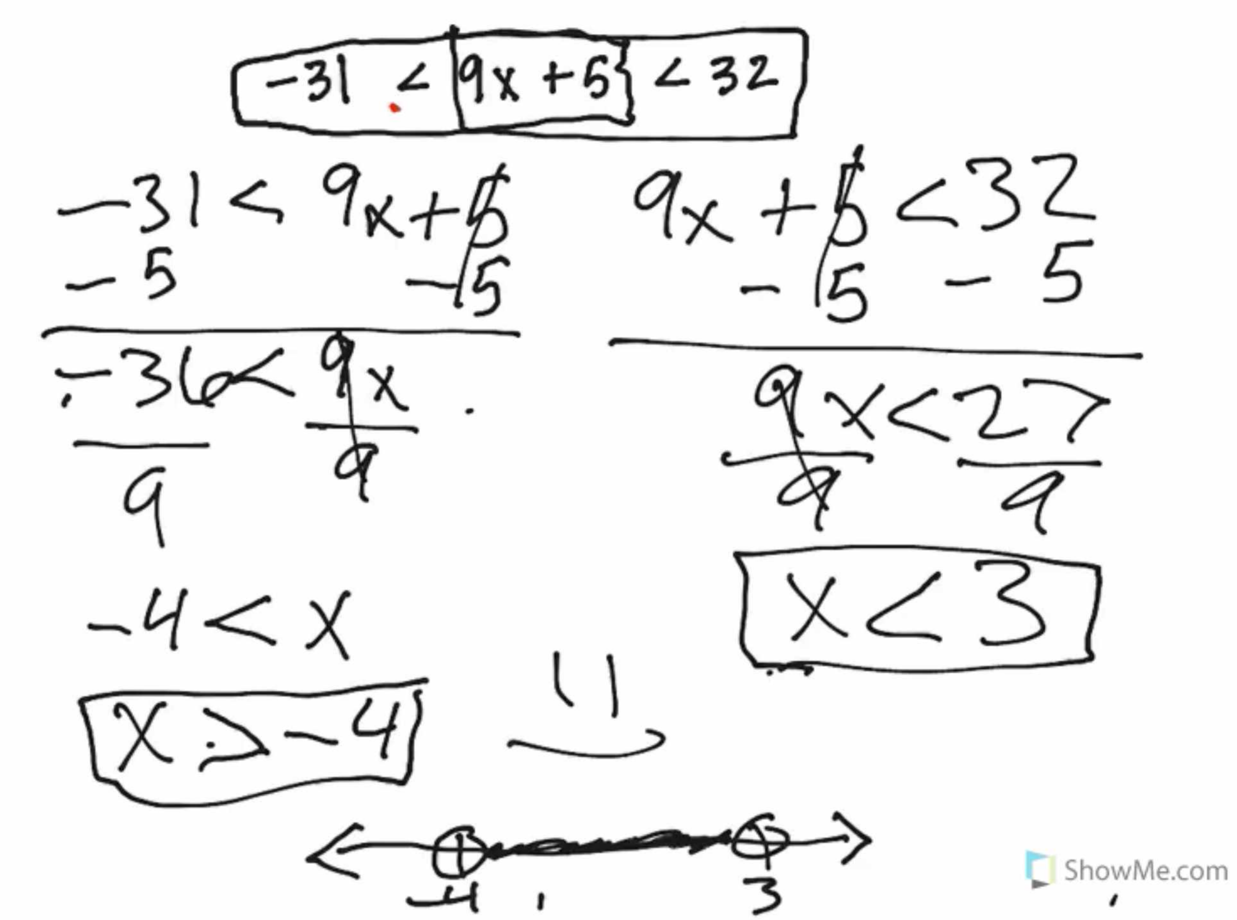
$$\cancel{+11} \quad +11 \qquad\qquad \cancel{+7} \quad +7$$

$$\frac{-4x}{-4} > \frac{16}{-4} \qquad\qquad \frac{8x}{8} > \frac{16}{8}$$

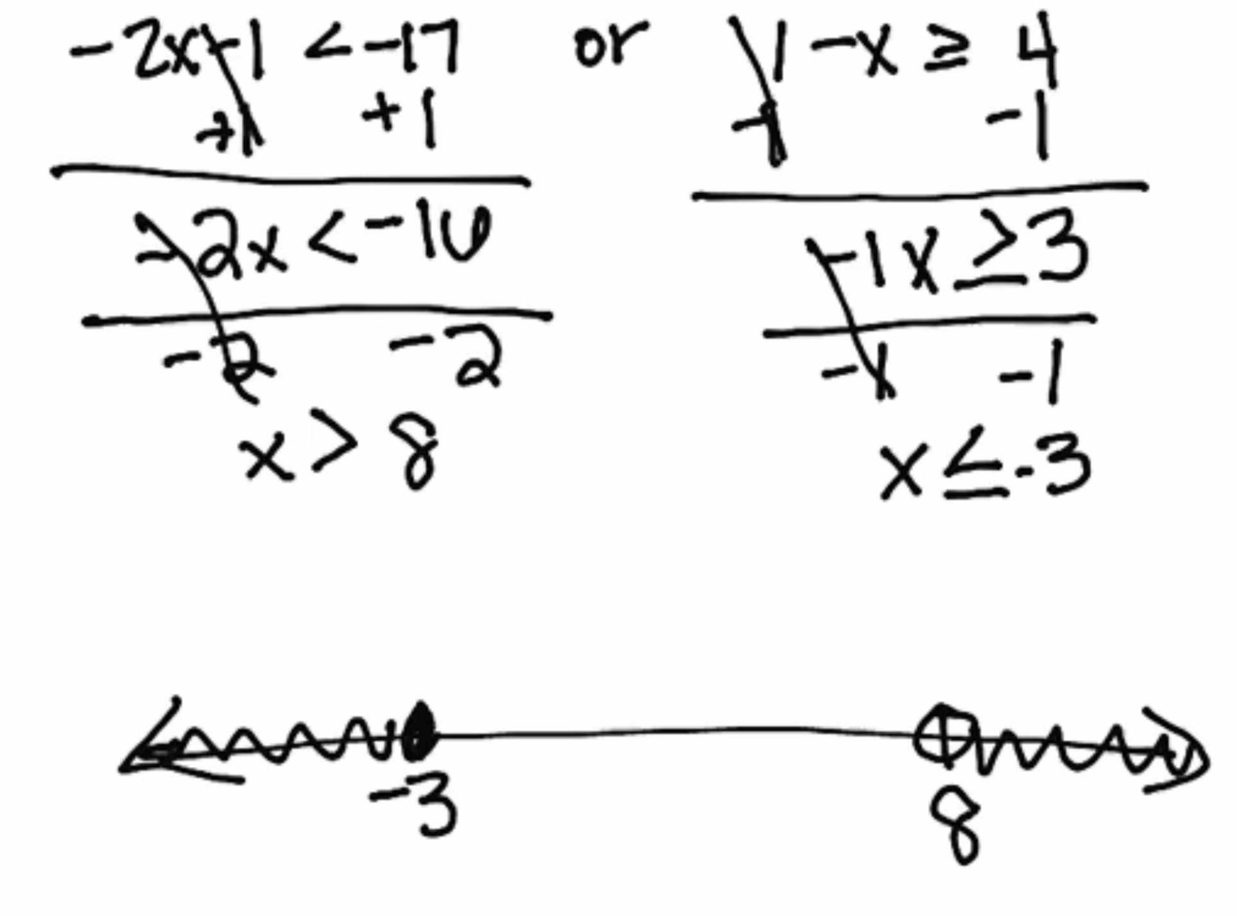
$$x < -4 \quad \text{or} \quad x > 2$$



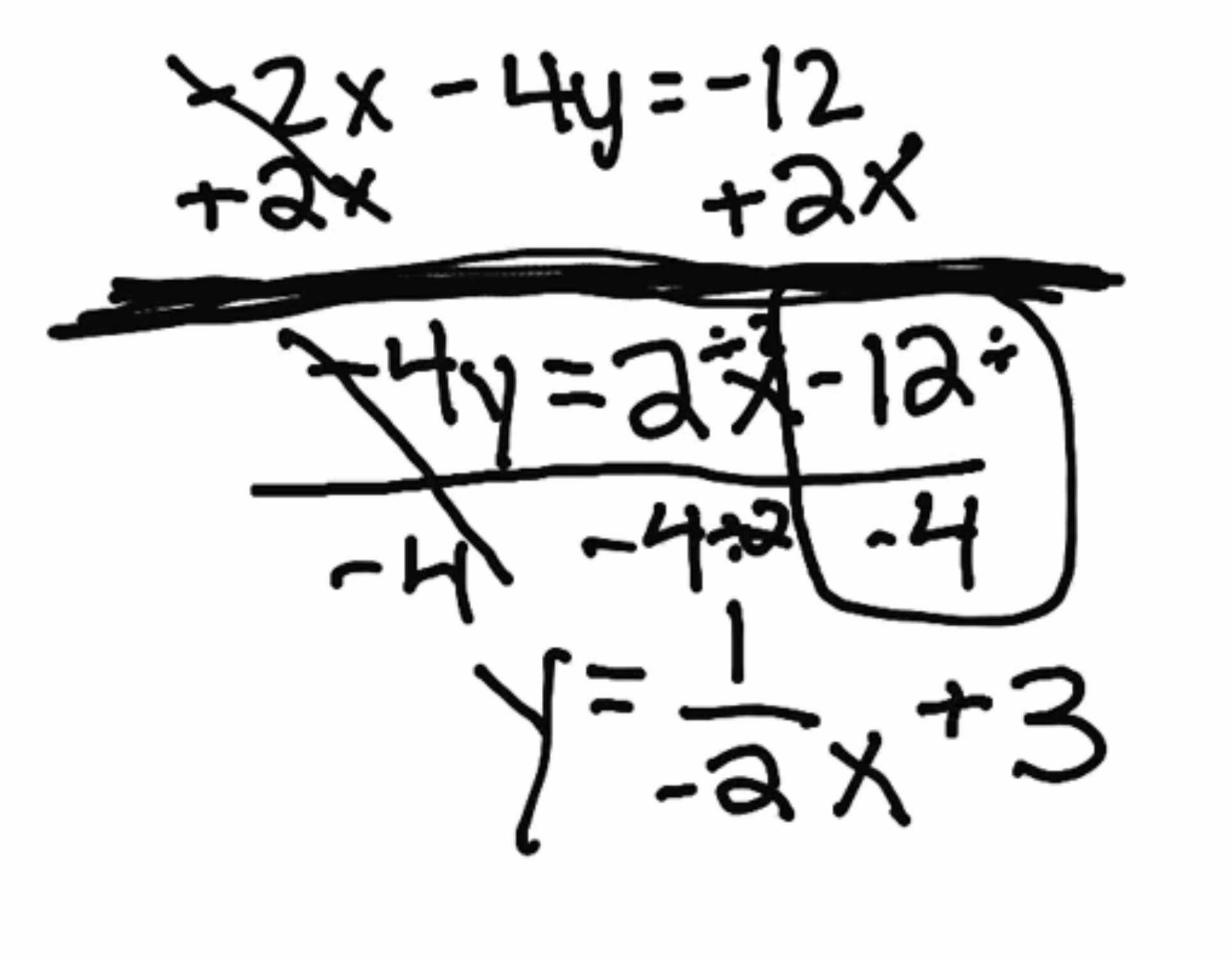
Appendix I: Cycle 2 – Apa Kua Solves a Compound Inequality



Appendix J: Cycle 2 – Ang Solves a Compound Inequality



Appendix K: Cycle 3 – Ang Solves for “y”



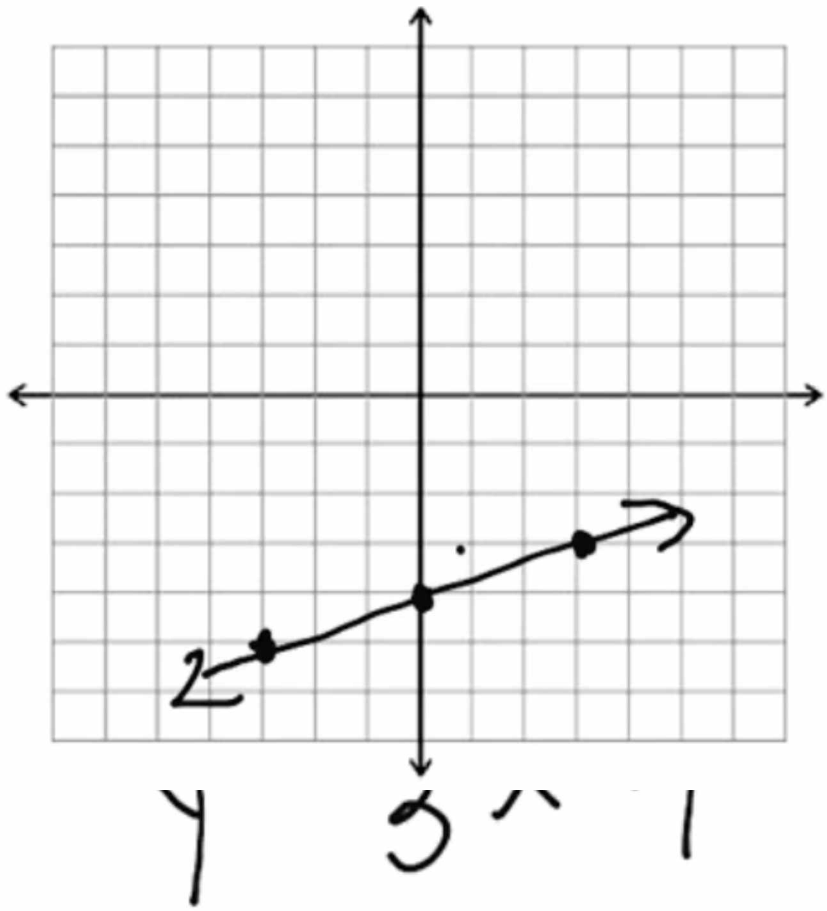
Appendix L: Cycle 3 – Apa Kua Solves for “y”

$$\cancel{x} - y = -8$$
$$\cancel{-x} \qquad -x$$

$$\frac{\cancel{-y}}{\cancel{-1}} = \frac{+x}{+1} \frac{+8}{+1}$$

$$y = x + 8$$

Appendix M: Cycle 4 – Ang Graphs a Line



Appendix N: Cycle 4 – Apa Kua Graphs a Line